Two-sets cut-uncut on planar graphs

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UNIVERSITY OF BERGEN

based on joint work with Matthias Bentert, Pål Grønås Drange, Fedor V. Fomin, and Petr A. Golovach

University of Warsaw Algorithms Seminar

19 May 2023

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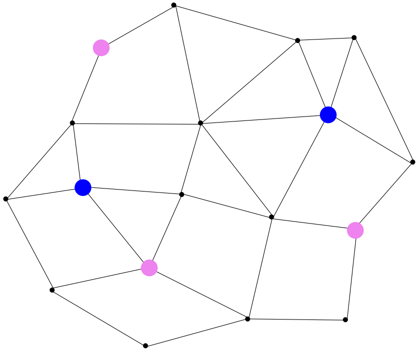
Output: Smallest number of edges to delete so that S is separated from T, but vertices in S, (resp. in T), stay in the same component.



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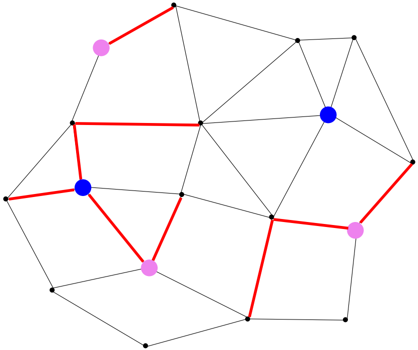

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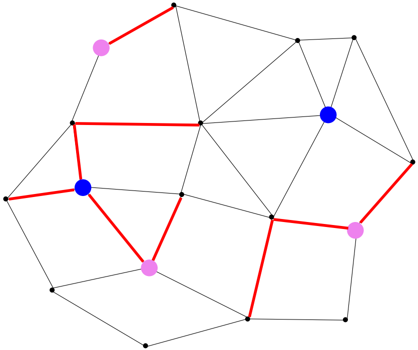

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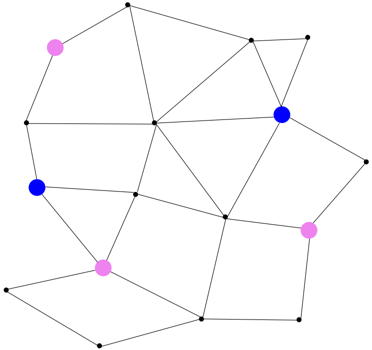
- Possible that no solution exists!



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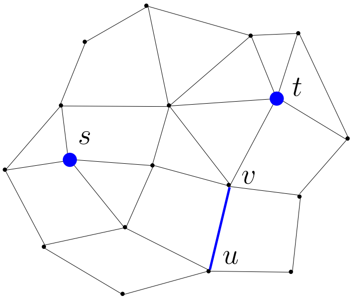
Output: Smallest number of edges to delete so that uv becomes an s-t-bridge.



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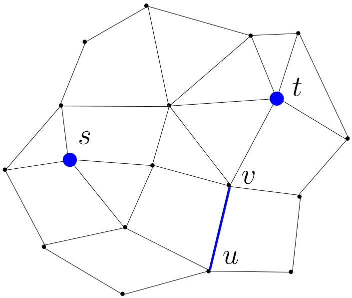

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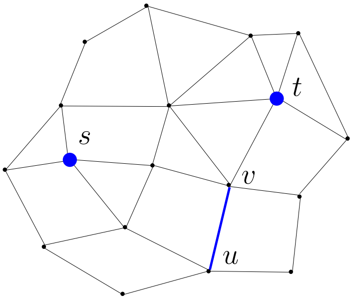

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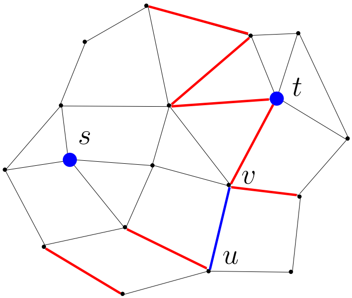

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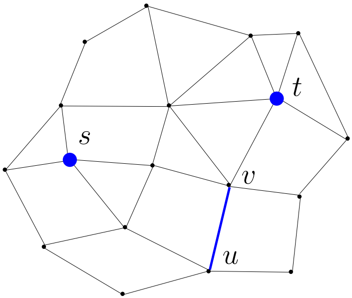

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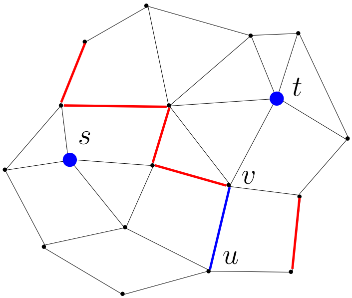

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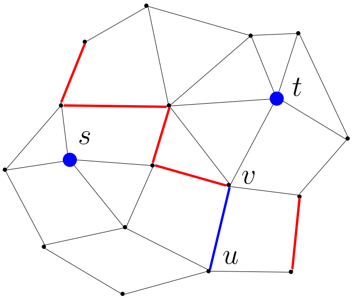

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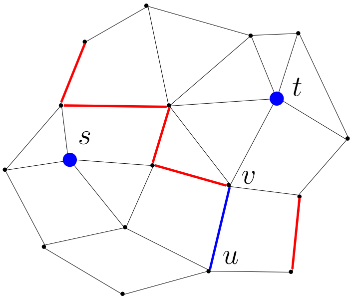

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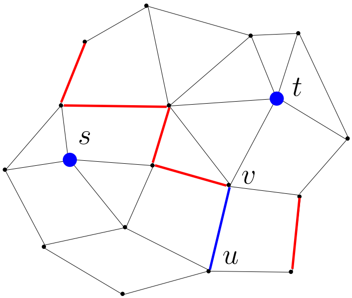

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Two-Sets Cut-Uncut can be solved on plane graphs in $4^{r+\mathcal{O}(\sqrt{r})}n^{\mathcal{O}(1)}$ time, where r is the minimum number of faces to cover $S \cup T$.

The algorithm

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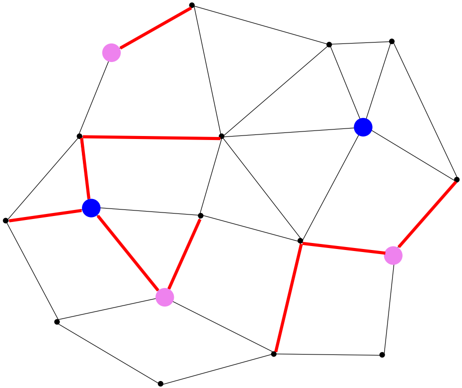

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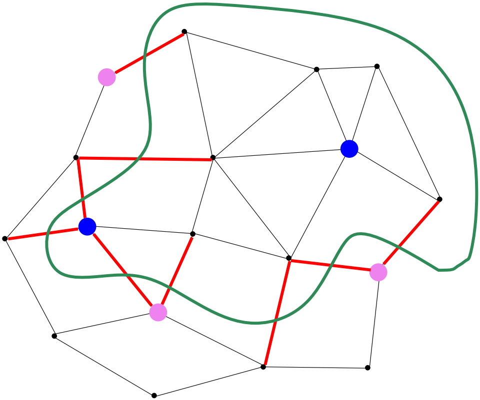

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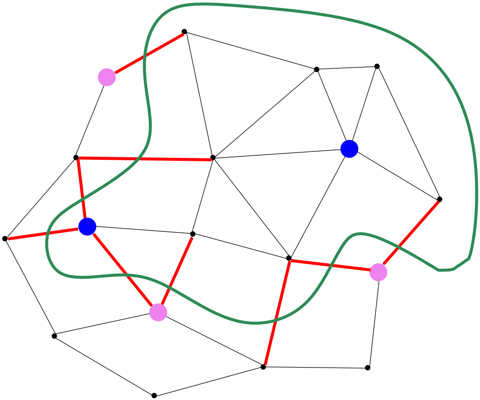

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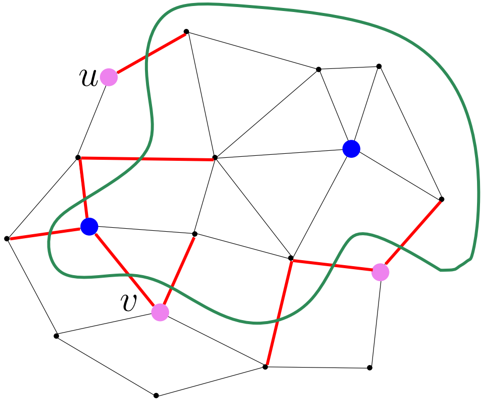

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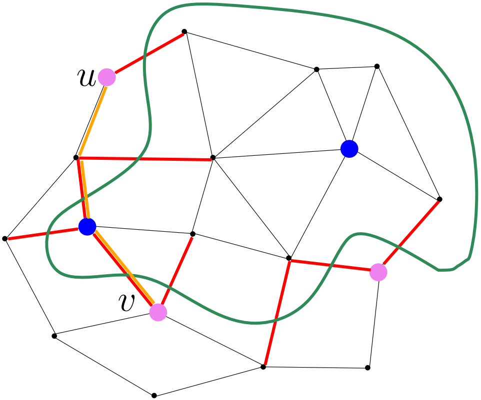

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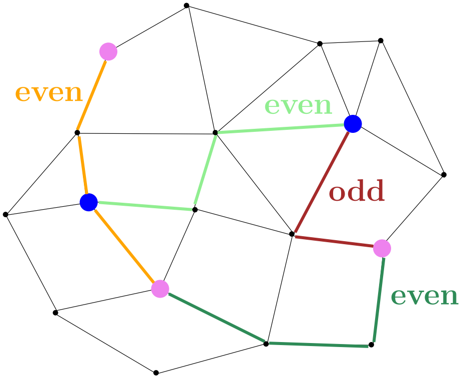

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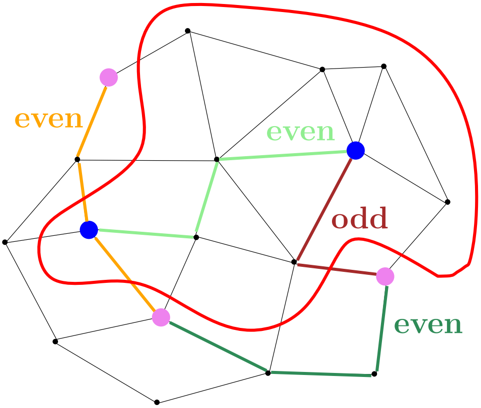

Shortest paths in group-labeled graphs

Boolean Group-Labeled Shortest Path

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- Generalizes the result of [Derigs'85] for $d = 1$
- Generalizes the result of [Björklund, Husfeldt, Taslaman '12] for finding a shortest cycle through T specified vertices in time $2^{|T|} n^{\mathcal{O}(1)}$

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The Algorithm for Boolean Group-Labeled Shortest Path

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$\Rightarrow$ By applying the Schwartz–Zippel lemma, the shortest solution can be found in randomized time $2^d n^{\mathcal{O}(1)}$

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- Non-zero if a solution of length ℓ exists, because a solution gives monomial corresponding to exactly one walk

The polynomial

- $P_\ell = \sum_{(v_0, v_1, \ldots, v_\ell) \in \mathcal{W}_\ell} \prod_{i=1}^{\ell} \mathbf{x}(v_{i-1} v_i)$ (over $\mathrm{GF}(2^{\lceil \log \ell \rceil + 2})$)
- Can evaluate in $2^d n^{\mathcal{O}(1)}$ time by dynamic programming over walks
- Non-zero if a solution of length ℓ exists, because a solution gives monomial corresponding to exactly one walk
- Remains to show that if no solutions of length $\leq \ell$ exists, then each monomial appears an even number of times

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  - Here we use that the group is $(\mathbb{Z}_2^d, +)$

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