Computing Treewidth

Tuukka Korhonen

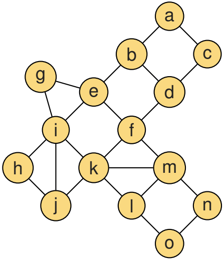
UNIVERSITY OF BERGEN

FPT Fest 2023 in the honor of Mike Fellows

15 June 2023

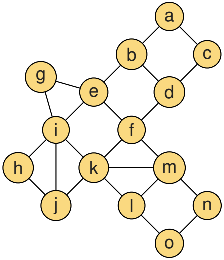
Treewidth

- Measures how close a graph is to a tree



Treewidth

- Measures how close a graph is to a tree
  - Trees have treewidth 1



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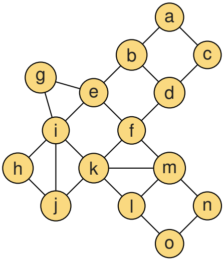

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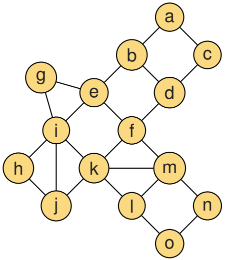

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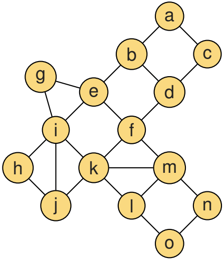

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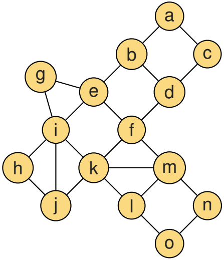

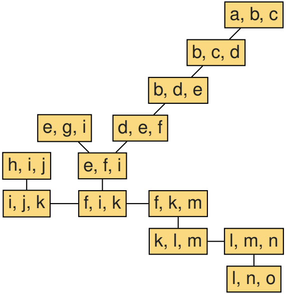

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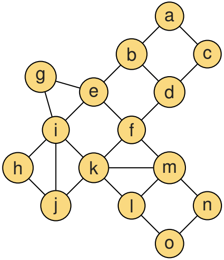

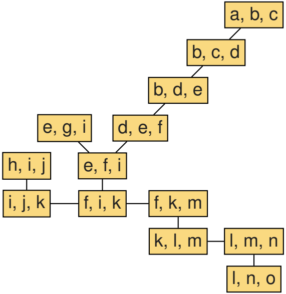

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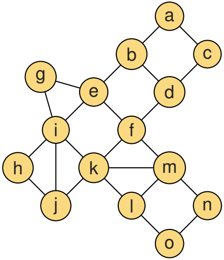

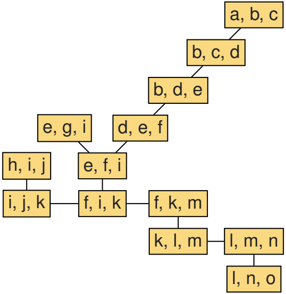

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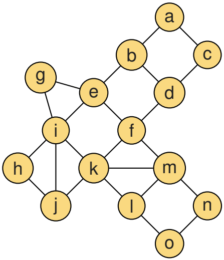

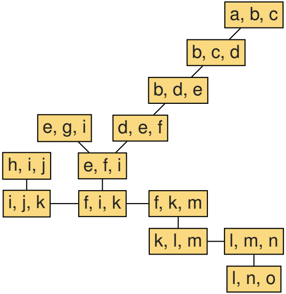

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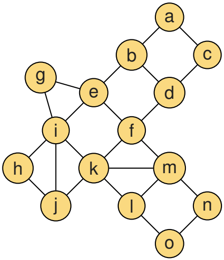

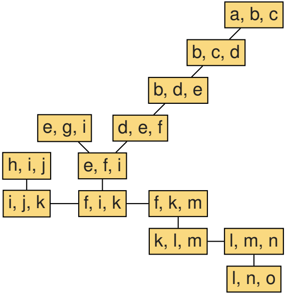

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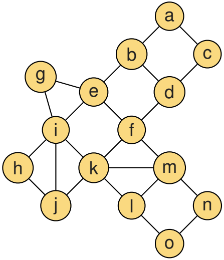

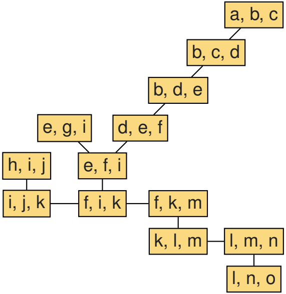

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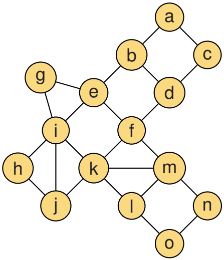
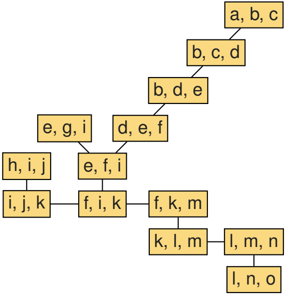
[Robertson & Seymour '84, Arnborg & Proskurowski '89, Bertele & Brioschi '72, Halin '76]





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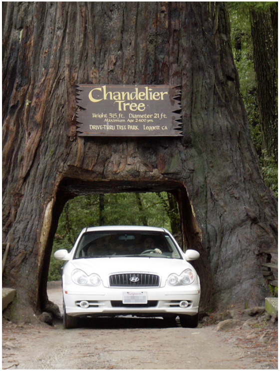
Why treewidth



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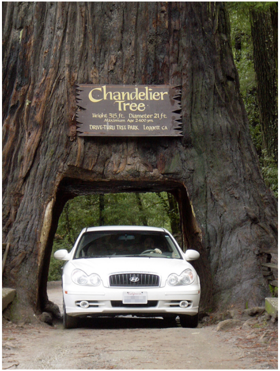

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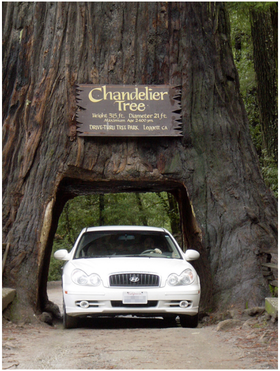

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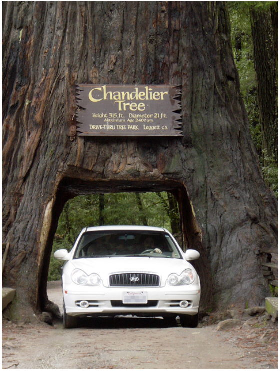

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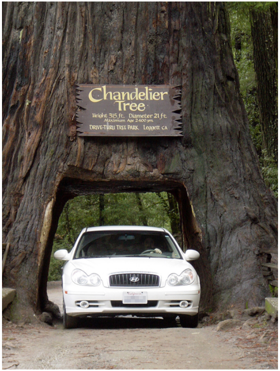
Need the tree decomposition!



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1. Robertson-Seymour FPT-approximation and its descendants

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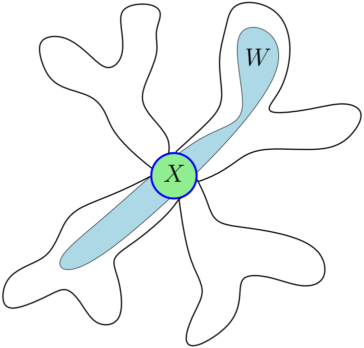
1. Robertson-Seymour FPT-approximation and its descendants
2. Classic exact FPT algorithms
3. New FPT algorithms based on local improvement

1. Robertson-Seymour FPT-approximation

Robertson-Seymour FPT-approximation: Balanced separators

Definition: Vertex set $X \subseteq V(G)$ is a balanced separator of a vertex set $W \subseteq V(G)$ if for every component C of $G - X$ it holds that $|W \cap C| \leq |W|/2$.



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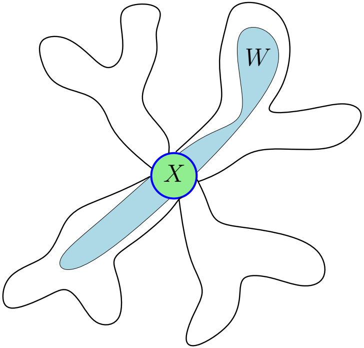

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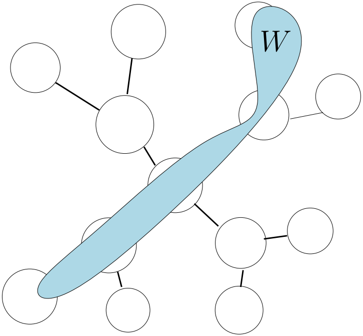

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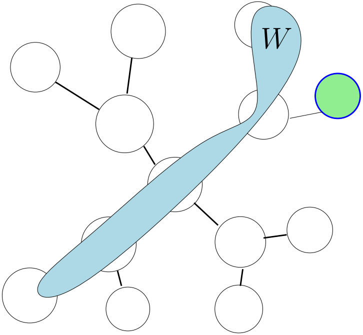

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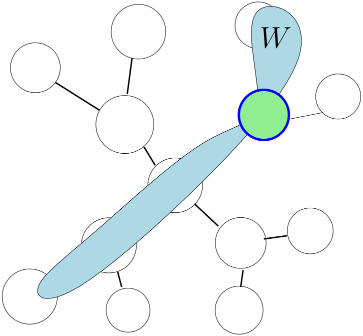

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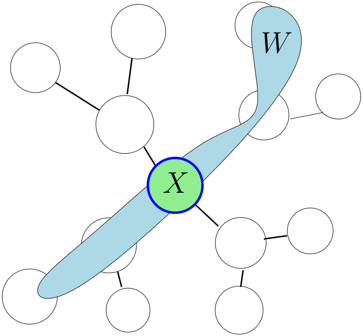

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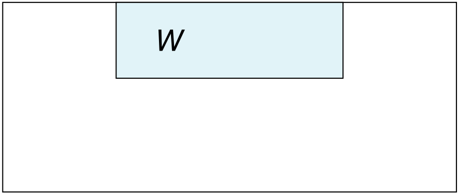

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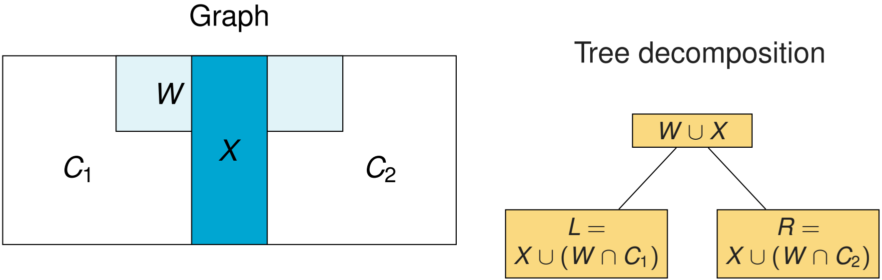

Balanced separator X with components C_1 and C_2

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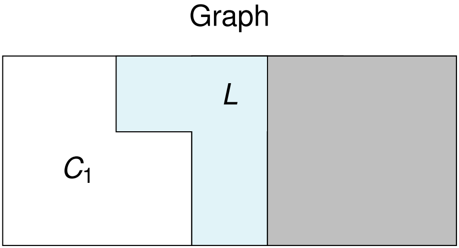

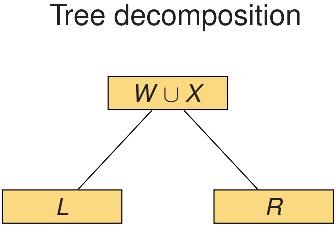

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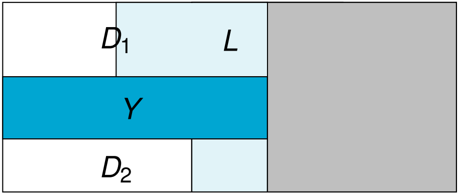

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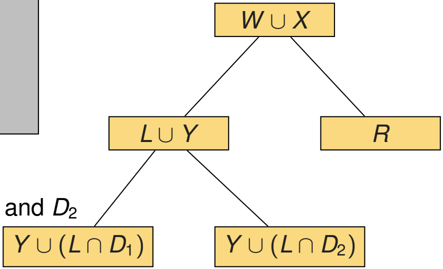

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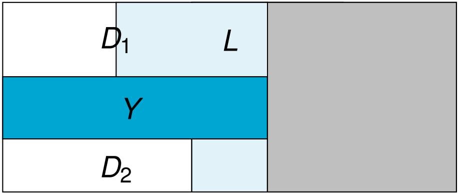

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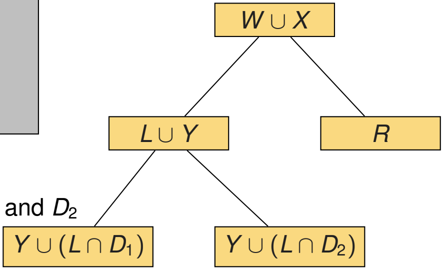

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Continue recursively...

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Theorem (Robertson & Seymour '86)

There is a $2^{\mathcal{O}(k)} n^2$ time 4-approximation for treewidth

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  - [Fomin, Lokshtanov, Pilipczuk, Saurabh & Wrochna'18]: $\mathcal{O}(k)$-approximation in time $k^{\mathcal{O}(1)} n \log n$
  - [Belbasi & Fürer '21]: 5-approximation in time $2^{7k} n \log n$

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- Idea applied to many other width parameters:

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- Idea applied to many other width parameters:
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  - And many more...

2. Classic exact FPT algorithms

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- Typical sequences applied to branchwidth [Bodlaender & Thilikos '97], cutwidth and carving-width [Thilikos, Serna & Bodlaender '00], rankwidth and matroid branchwidth [Jeong, Kim & Oum '18], and more...

3. New FPT algorithms based on local improvement

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- Techniques extended also to 2-approximating branchwidth in time $2^{\mathcal{O}(k)} n$ and rankwidth in time $2^{2^{\mathcal{O}(k)}} n^2$ [Fomin & K. '22]

FPT 2-approximation: Outline

By the recursive compression technique of [Bodlaender '93] we can focus on:

Input: Graph G and a tree decomposition of G of width w

Output: A tree decomposition of G of width $< w$ or conclusion that $w \leq 2 \cdot \text{tw}(G) + 1$

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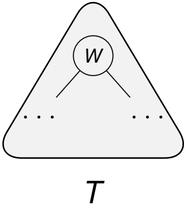
   - Efficient implementation by amortizatized analysis of the improvements and dynamic programming over the tree decomposition

The improvement operation

- Let W be a largest bag



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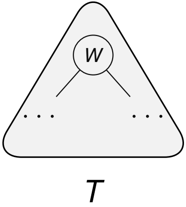

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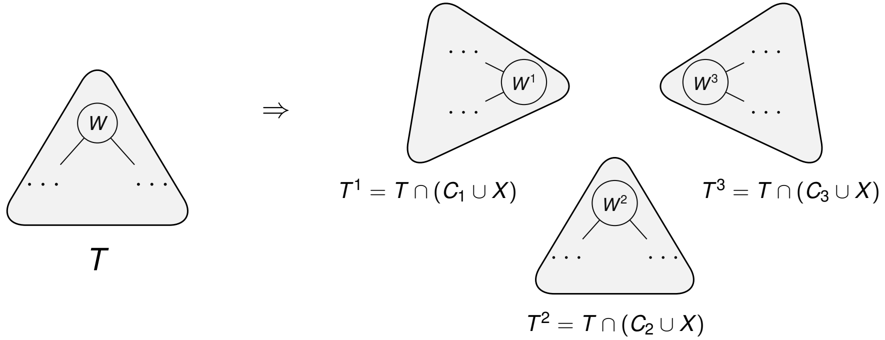

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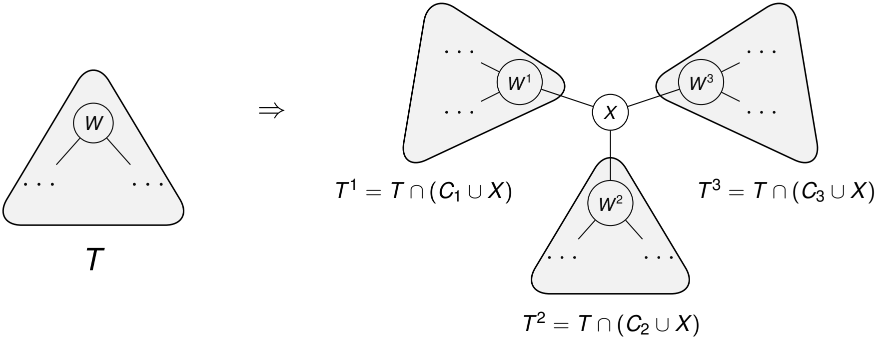

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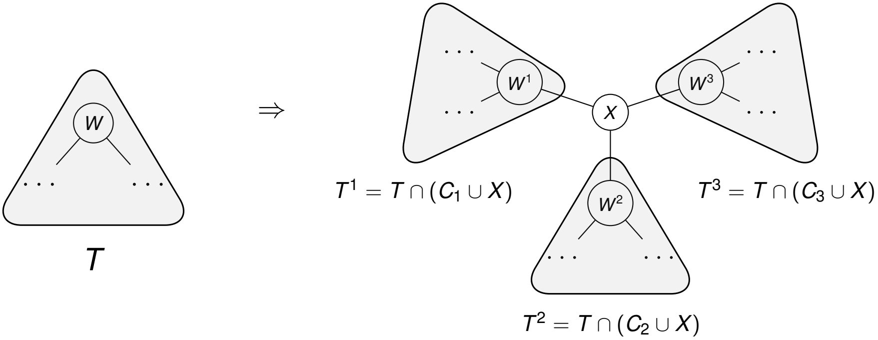

Except that vertices in X may violate the connectedness condition

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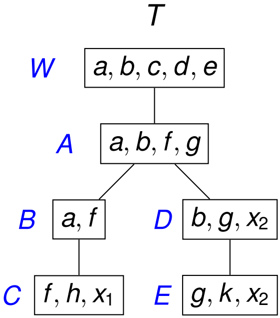

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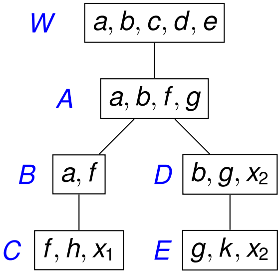
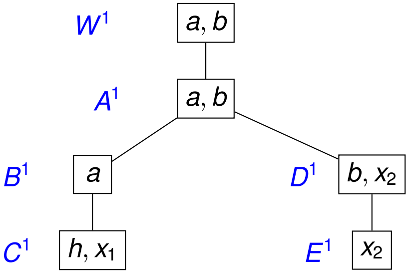


$T^1 = T \cap (C_1 \cup X)$



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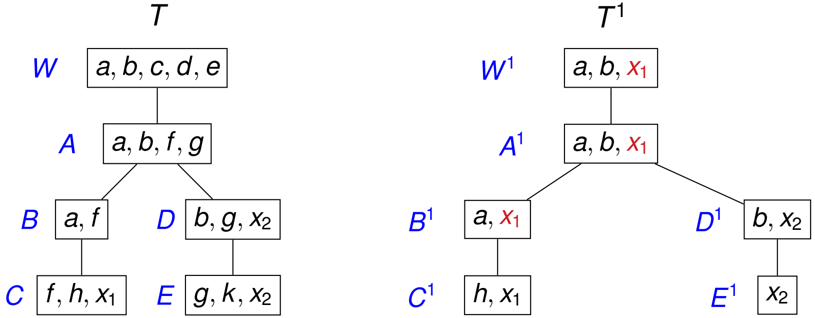

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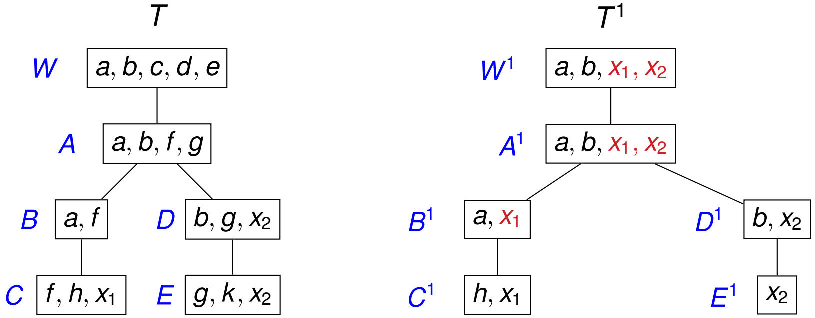

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$\Rightarrow$ The number of bags of size $|W|$ decreases

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- Same idea of improving a tree decomposition by decreasing the number of largest bags

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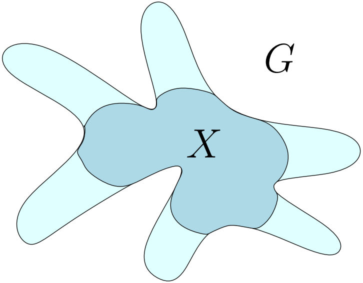

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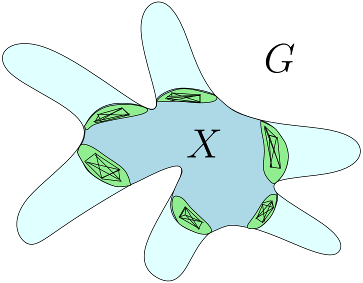

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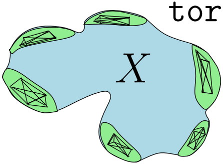

- Make neighborhoods of components of $G - X$ into cliques
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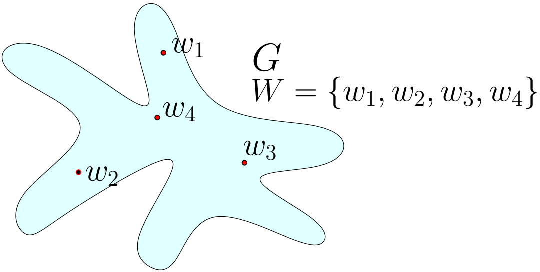

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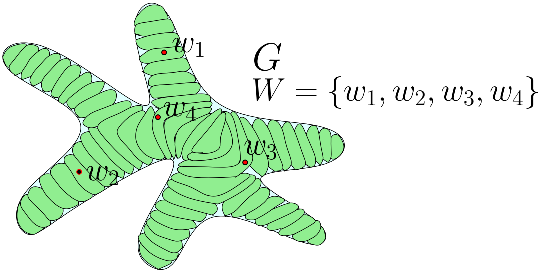

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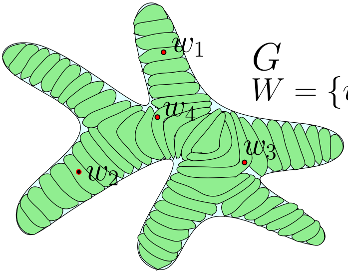

G

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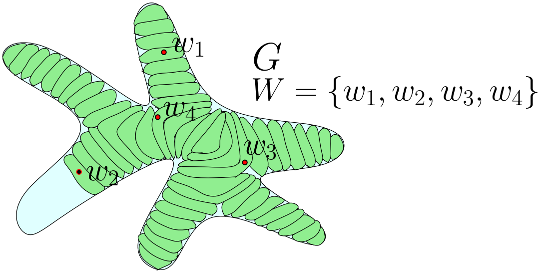

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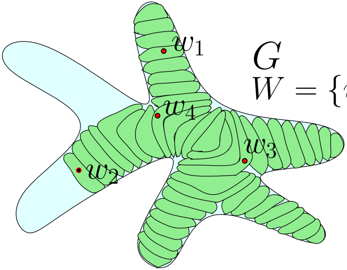

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Improving a tree decomposition

Let W be a largest bag of T SUBSET TREEWIDTH

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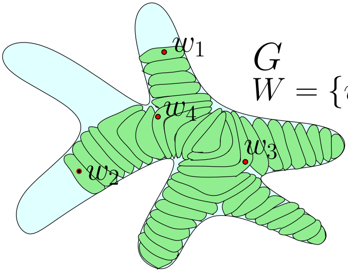

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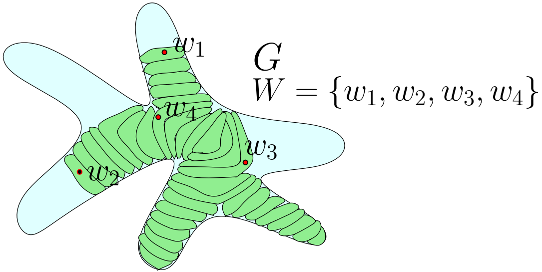

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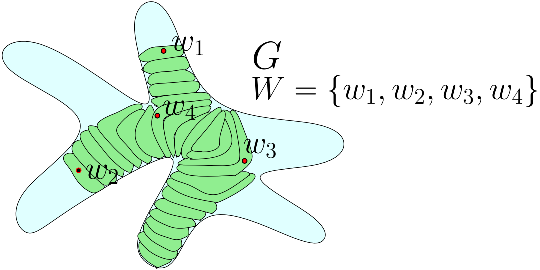

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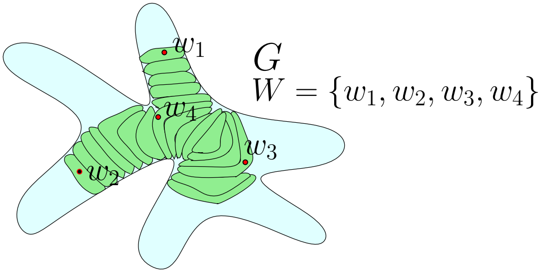

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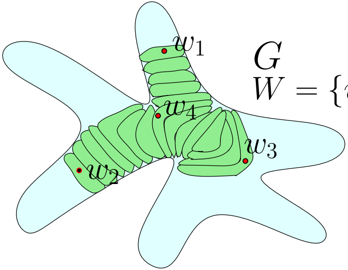

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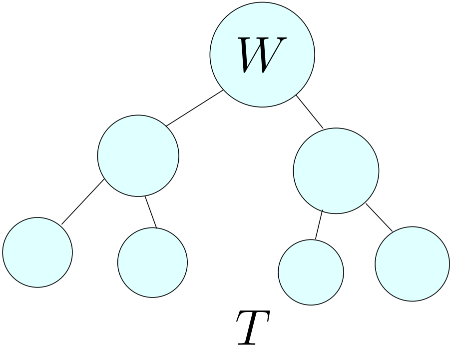

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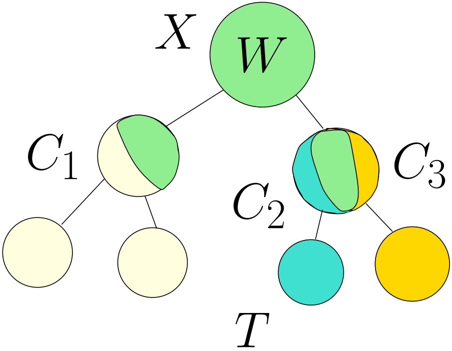

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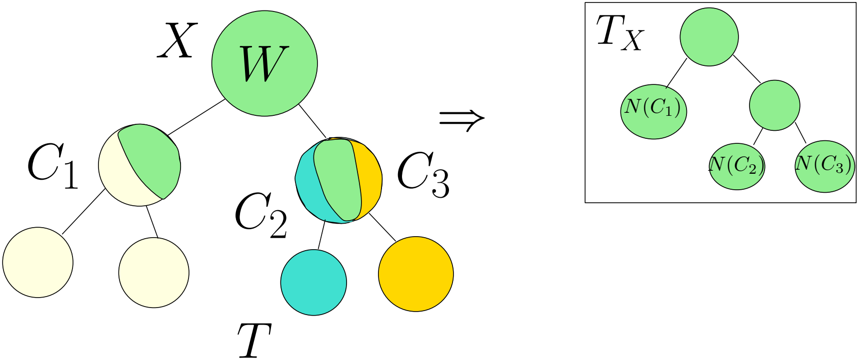

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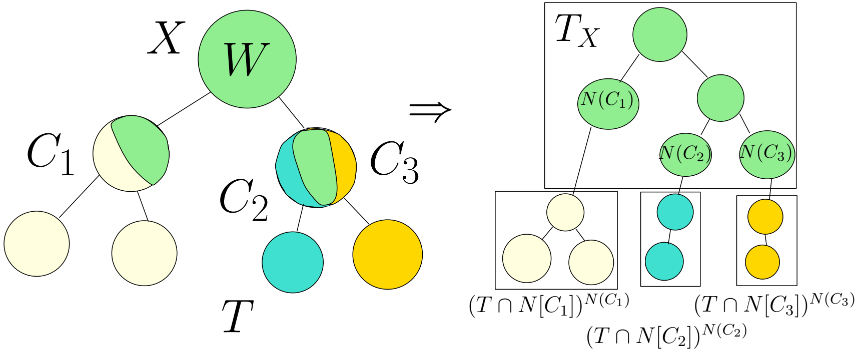

Subset treewidth for exact FPT algorithms

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Input: Graph G, integer k, set of vertices $W \subseteq V(G)$ with $|W| = k + 2$

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Subset treewidth for FPT-approximation

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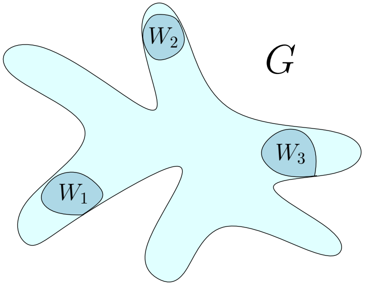

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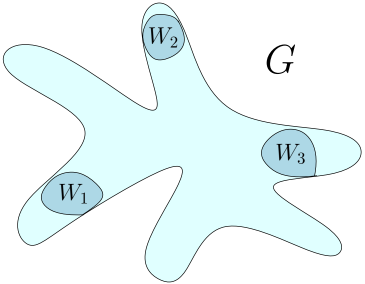

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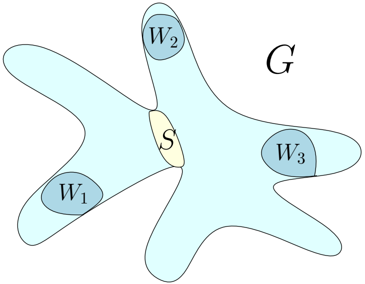

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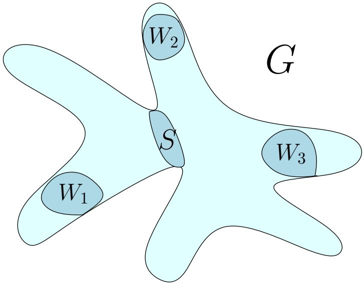

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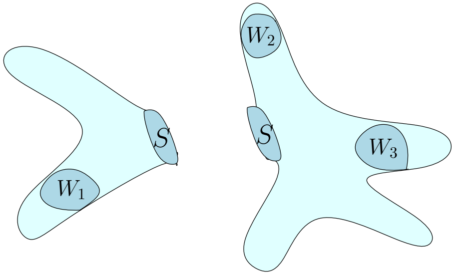

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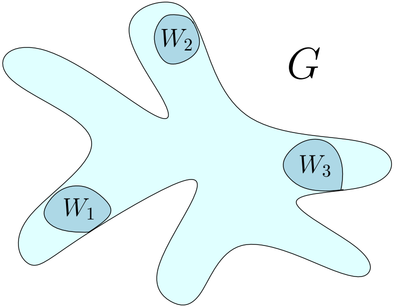

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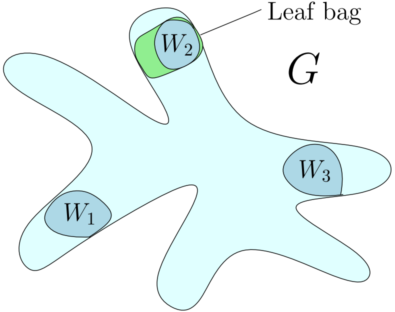

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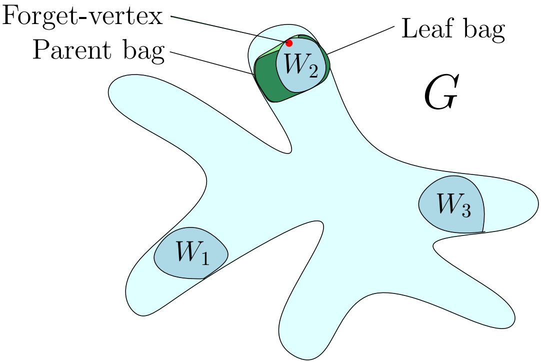

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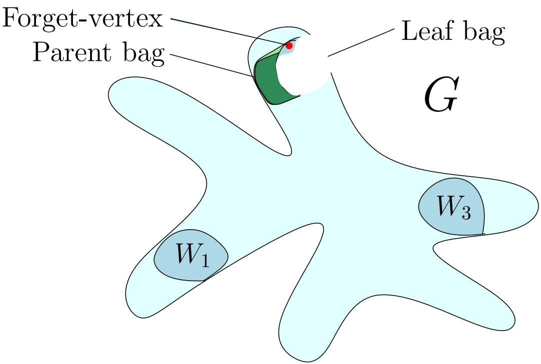

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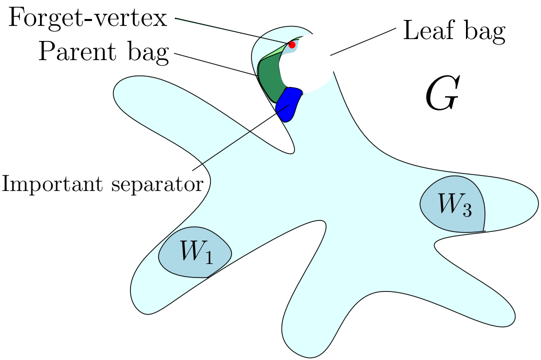

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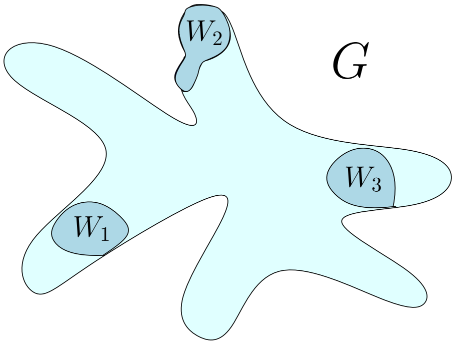

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- Improve either dependence on k or n in the $2^{\mathcal{O}(k^2)} n^4$ exact treewidth algorithm

Thank you!