SharpSAT-TD: Improving SharpSAT by Exploiting Tree Decompositions

Tuukka Korhonen and Matti Järvisalo

HIIT, Department of Computer Science, University of Helsinki, Finland

MC-2021 Online July 6, 2021



SharpSAT-TD

- New modification of SharpSAT [Thurley '06]
 - 1. Integrates low-width tree decompositions to the variable selection heuristic
 - 2. Implements new preprocessor
 - 3. Directly supports weighted model counting

MCC-2021 Results on Public Instances

solver	config	solved	solver	config	solved	solver	config	solved
\$	\$	•	\$	\$		\$	\$	•
sharp-tw- unweighte d 고	default 🖒	83/100 பீ	sharp-tw- weighted	default 🖒	99/100 리	Narsimha ♂	track4_co nf2.sh ⊡	69/100 Ľ
Narsimha ப	track1_co nf2.sh ⊠	69/100 ෆ්				sharp-tw- unweighte	default 🗗	69/100 பீ
			d4 ⊡"	TRACK2+ 3ds prepr	81/100 고	dc²		
Narsimha 戊	track1_co nf1.sh ⊡	61/100 ⊡		ocEquiv.s h ⊠		Narsimha பி	track4_co nf1.sh ⊡	68/100 ්
d4 ය	TRACK1+ 4_ds_pre procShar pEquiv.sh 岱	59/100 ⊠	d4 ⊡	TRACK2+ 3_ms_pre procEquiv .sh ⊡	81/100 ෆ්	d4 ⊡ੋ	TRACK1+ 4_ds_pre procShar pEquiv.sh	57/100 대
			c2d ⊡"	default 🗗	74/100 ௴		ĊĊ	
d4 🖒	TRACK1+ 4_ms_pre procShar pEquiv.sh ⊠	57/100 ය				d4 앱 TRACK1+ 4_ms_pre procShar pEquiv.sh 岱	57/100 岱	
			Narsimha		72/100 더			
TwG ⊡	2.sh ₫	38/100				dpmc4fix ⊡	4 🖒	49/100 고

Overview

Overview of SharpSAT-TD

- 1. Preprocess
- 2. Compute a tree decomposition with FlowCutter [Strasser '17]
- 3. Count using tree decomposition guided variable selection

Overview

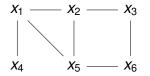
Overview of SharpSAT-TD

- 1. Preprocess
- 2. Compute a tree decomposition with FlowCutter [Strasser '17]
- 3. Count using tree decomposition guided variable selection

I will first talk about (3), then about (1), and then about other changes compared to SharpSAT

Tree Decompositions

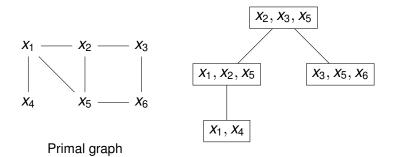
$$(\neg x_2 \lor x_3) \land (x_3 \lor \neg x_6) \land (x_5 \lor x_6) \land (x_1 \lor \neg x_2 \lor x_5) \land (x_1 \lor \neg x_4)$$



Primal graph

Tree Decompositions

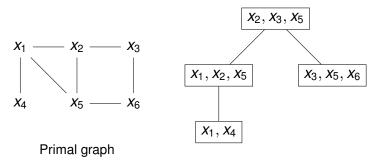
$$(\neg x_2 \lor x_3) \land (x_3 \lor \neg x_6) \land (x_5 \lor x_6) \land (x_1 \lor \neg x_2 \lor x_5) \land (x_1 \lor \neg x_4)$$



Tree decomposition

Tree Decompositions

$$(\neg x_2 \lor x_3) \land (x_3 \lor \neg x_6) \land (x_5 \lor x_6) \land (x_1 \lor \neg x_2 \lor x_5) \land (x_1 \lor \neg x_4)$$

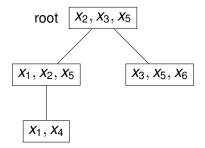


Tree decomposition

- Width of a tree decomposition: Size of the largest bag -1
- Treewidth of a graph/CNF: Minimum width of a tree decomposition

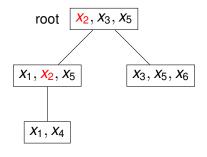
 Select the variable of the active formula that appears the closest to the root in the tree decomposition

 $(\neg x_2 \lor x_3) \land (x_3 \lor \neg x_6) \land (x_5 \lor x_6) \land (x_1 \lor \neg x_2 \lor x_5) \land (x_1 \lor \neg x_4)$



 Select the variable of the active formula that appears the closest to the root in the tree decomposition

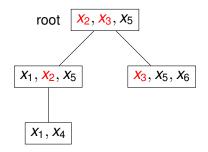
$$(x_3) \wedge (x_3 \vee \neg x_6) \wedge (x_5 \vee x_6) \wedge (x_1 \vee x_5) \wedge (x_1 \vee \neg x_4)$$



$$x_2 = 1$$
,

 Select the variable of the active formula that appears the closest to the root in the tree decomposition

$$(x_5 \lor x_6) \land (x_1 \lor \neg x_2 \lor x_5) \land (x_1 \lor \neg x_4)$$



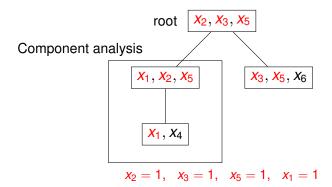
$$x_2 = 1, x_3 = 1,$$

 Select the variable of the active formula that appears the closest to the root in the tree decomposition

 $(x_1 \vee \neg x_4)$

*X*₂, *X*₃, *X*₅ root Component analysis X_1, X_2, X_5 X_3, X_5, X_6 X_1, X_4 $x_2 = 1$, $x_3 = 1$, $x_5 = 1$,

 Select the variable of the active formula that appears the closest to the root in the tree decomposition



Theoretical Background

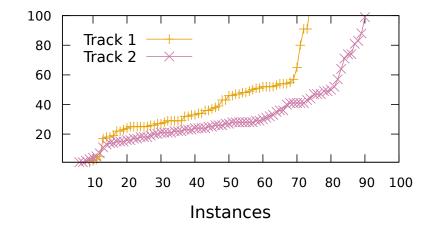
Proposition ([BDP03, Dar01])

Standard #DPLL algorithm, with component analysis and component caching, works in 2^{w} poly($|\phi|$) time when using a tree decomposition of width *w* for variable selection.

Theoretical Background

Proposition ([BDP03, Dar01])

Standard #DPLL algorithm, with component analysis and component caching, works in 2^{w} poly($|\phi|$) time when using a tree decomposition of width *w* for variable selection.



Width

Variable x with highest score(x) is selected.

Standard SharpSAT:

$$score(x) = act(x) + freq(x)$$

- act(x) is VSIDS-like activity score
- freq(x) is the number of occurrences of x in the current formula

Variable x with highest score(x) is selected.

Standard SharpSAT:

$$score(x) = act(x) + freq(x)$$

SharpSAT-TD:

$$score(x) = act(x) + freq(x) - C \cdot d(x)$$

- act(x) is VSIDS-like activity score
- freq(x) is the number of occurrences of x in the current formula
- d(x) is the distance from root of tree decomposition to closest bag containing x
- C is some positive constant

Variable x with highest score(x) is selected.

Standard SharpSAT:

$$score(x) = act(x) + freq(x)$$

SharpSAT-TD:

$$score(x) = act(x) + freq(x) - C \cdot d(x)$$

- act(x) is VSIDS-like activity score
- freq(x) is the number of occurrences of x in the current formula
- d(x) is the distance from root of tree decomposition to closest bag containing x
- C is some positive constant
 - ▶ If *C* is large, selection is purely by tree decomposition
 - If C is small, selection is same as in standard SharpSAT

Variable x with highest score(x) is selected.

Standard SharpSAT:

$$score(x) = act(x) + freq(x)$$

SharpSAT-TD:

$$score(x) = act(x) + freq(x) - C \cdot d(x)$$

- act(x) is VSIDS-like activity score
- freq(x) is the number of occurrences of x in the current formula
- d(x) is the distance from root of tree decomposition to closest bag containing x
- C is some positive constant
 - ▶ If *C* is large, selection is purely by tree decomposition
 - If C is small, selection is same as in standard SharpSAT
 - C chosen per-instance based on the width of the tree decomposition

Preprocessing

1. Complete vivification (minimalize each clause, with SAT solver)

- 1. Complete vivification (minimalize each clause, with SAT solver)
- 2. Redundant clause deletion

- 1. Complete vivification (minimalize each clause, with SAT solver)
- 2. Redundant clause deletion
- 3. Equivalent variable merging (treewidth-aware)

- 1. Complete vivification (minimalize each clause, with SAT solver)
- 2. Redundant clause deletion
- 3. Equivalent variable merging (treewidth-aware)
- 4. Re-implementation of B+E [LLM16] (treewidth-aware)

Other Modifications

- "Implicit BCP" disabled
- LBD learned clause scoring scheme [AS09]
- Probabilistic component caching [SRSM19]

- "Implicit BCP" disabled
- LBD learned clause scoring scheme [AS09]
- Probabilistic component caching [SRSM19]
- Extension to weighted model counting via template parameters easily extensible to model counting over any semiring

Thank you for your attention!

Bibliography

Gilles Audemard and Laurent Simon.

Predicting learnt clauses quality in modern SAT solvers. In *IJCAI*, pages 399–404, 2009.

F. Bacchus, S. Dalmao, and T. Pitassi.

Algorithms and complexity results for #SAT and Bayesian inference. In FOCS, pages 340–351. IEEE, 2003.



A. Darwiche.

Decomposable negation normal form. J. ACM, 48(4):608–647, 2001.



J. Lagniez, E. Lonca, and P. Marquis.

Improving model counting by leveraging definability. In IJCAI, pages 751–757. IJCAI/AAAI Press, 2016.



S. Sharma, S. Roy, M. Soos, and K. S. Meel.

GANAK: A scalable probabilistic exact model counter. In *IJCAI*, pages 1169–1176. ijcai.org, 2019.