Introduction to theory of computing via graph coloring

Tuukka Korhonen

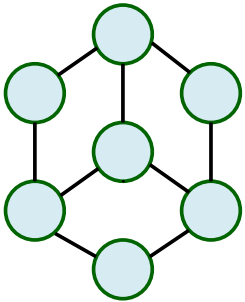
16 April 2026

IUP Lecture

Graphs and colorings

Graphs and colorings

- A graph consist of nodes (circles) connected by edges (line segments)



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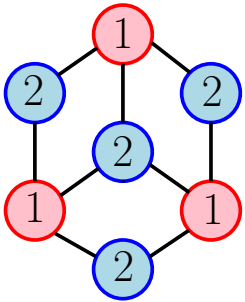

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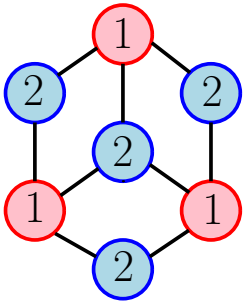

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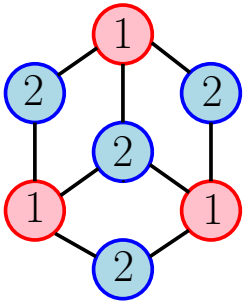
- Exercise 1: Consider the given graphs a-d, and find 2-colorings or that 2-coloring is not possible



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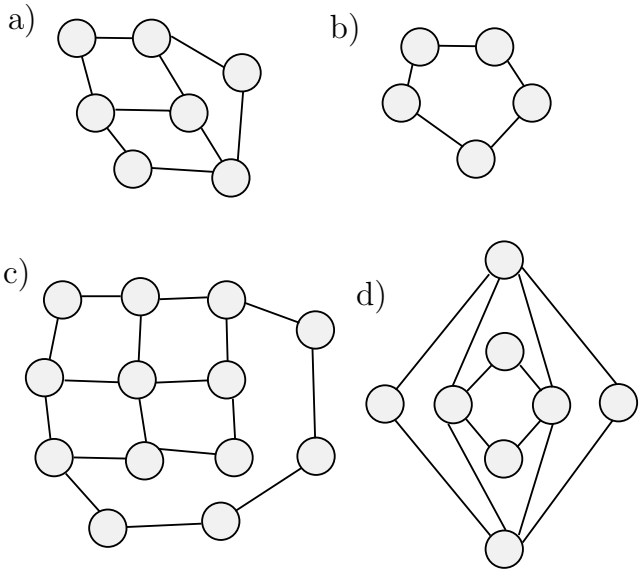

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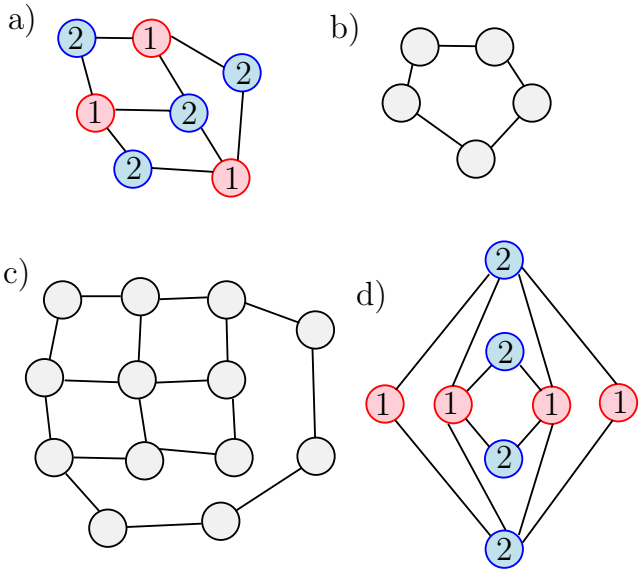

Algorithm for 2-coloring

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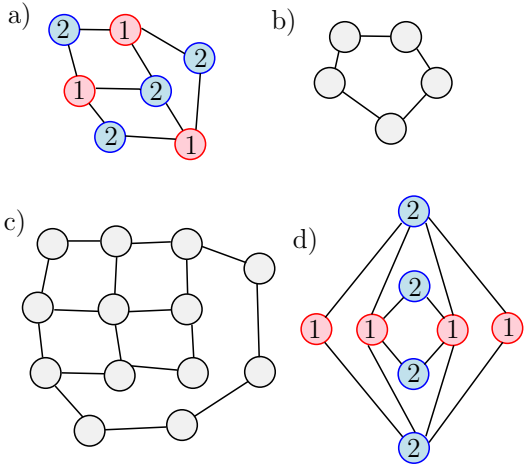

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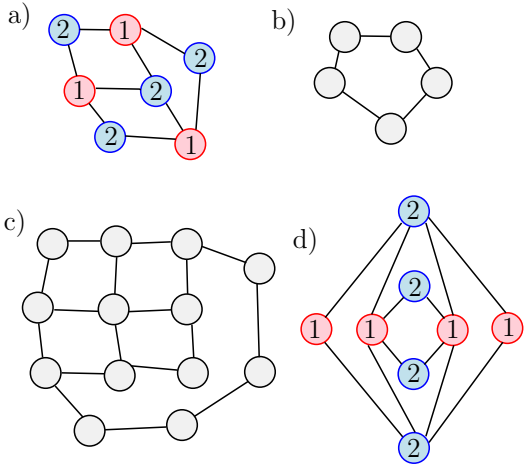

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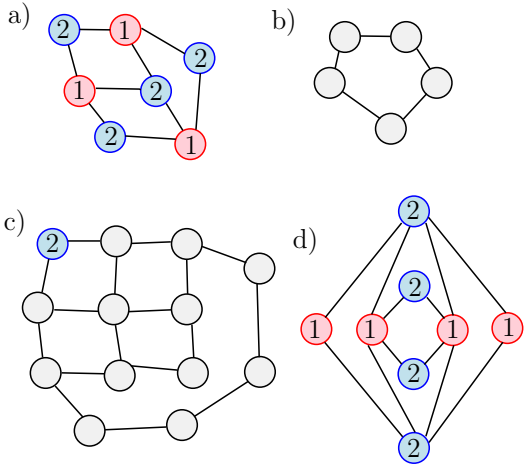

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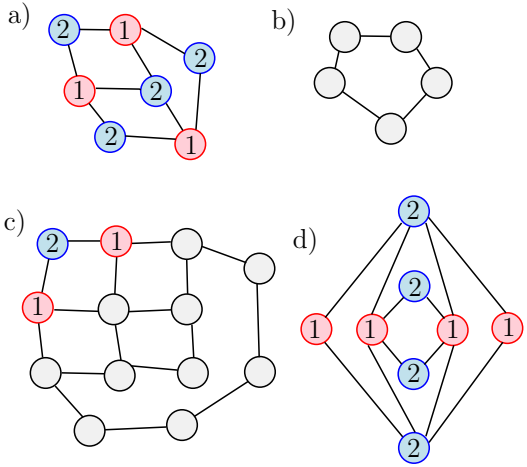

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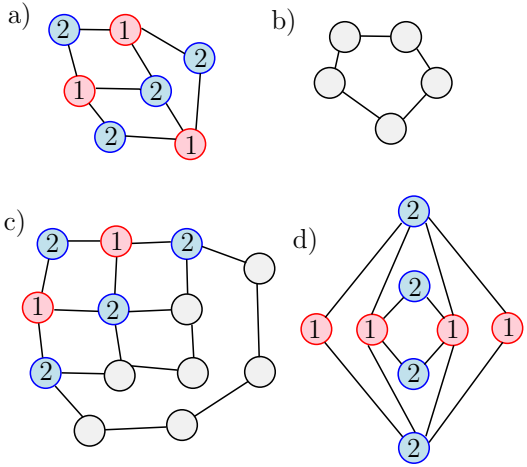

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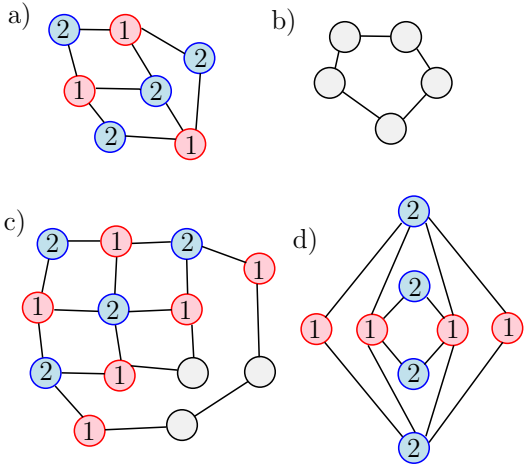

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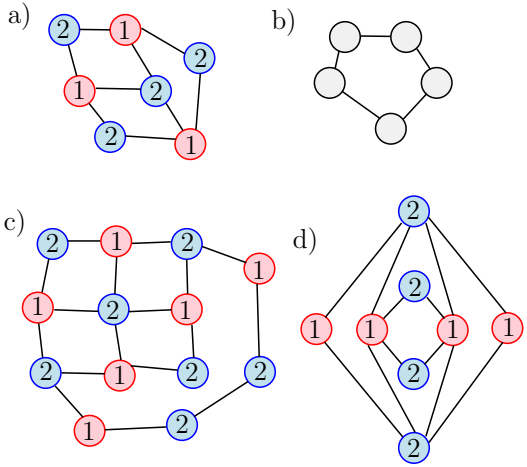

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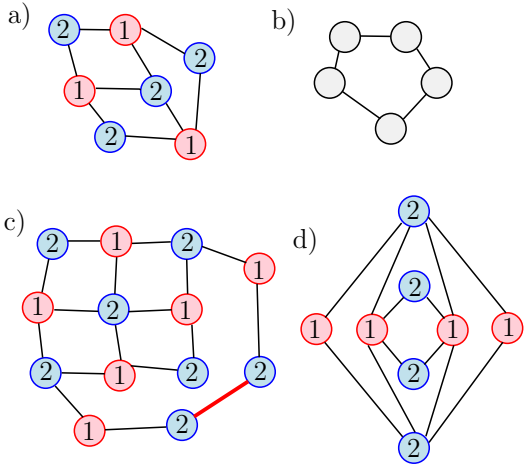

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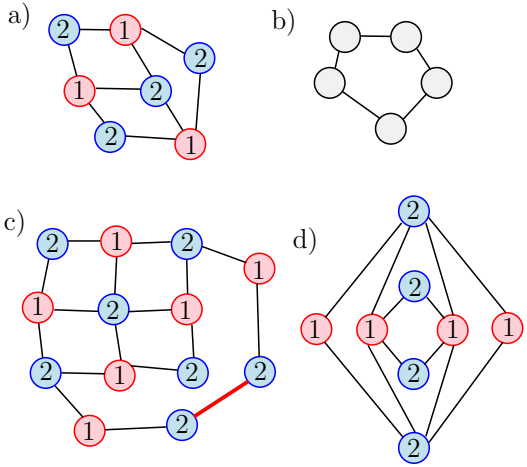

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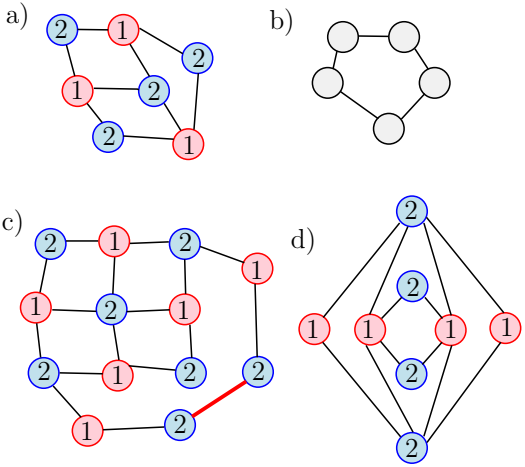
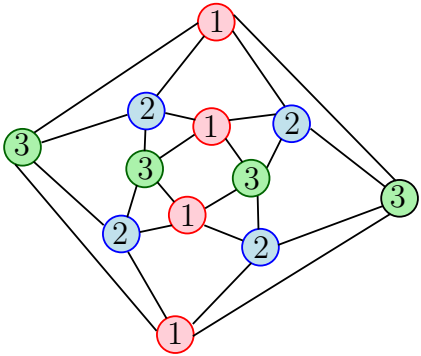
- Runs in $\approx n + m$ steps, where n the number of nodes, and m the number of edges



3-coloring

- How about 3-coloring?



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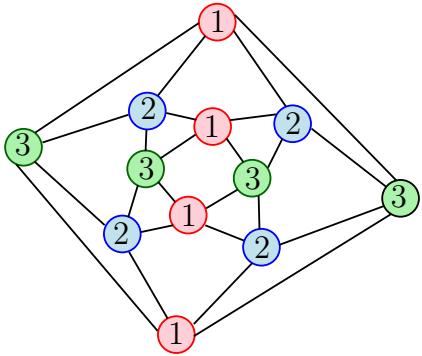

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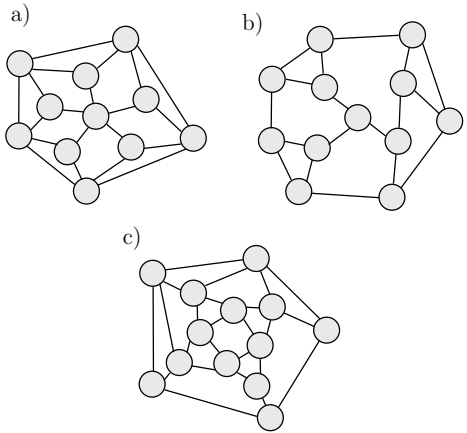

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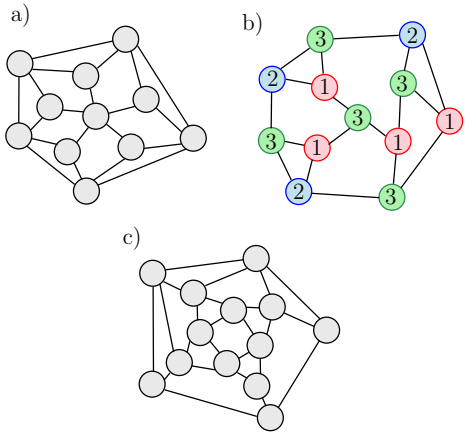

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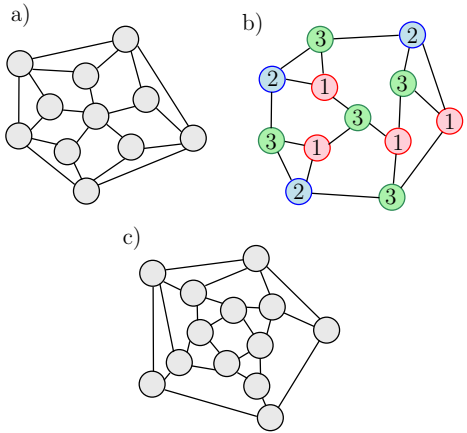

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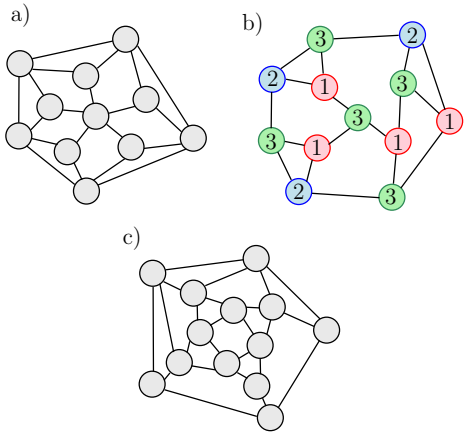

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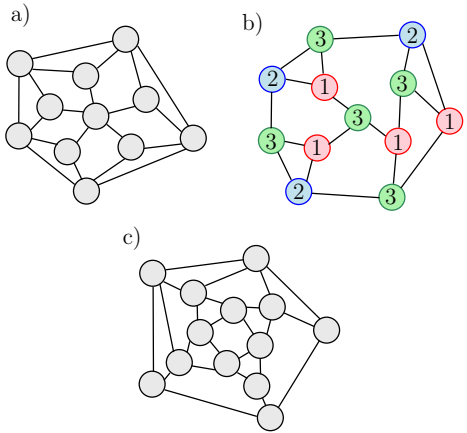

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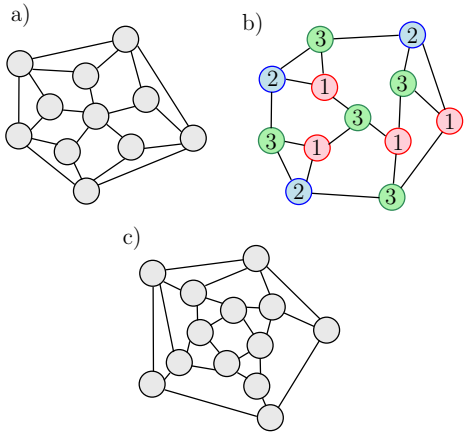

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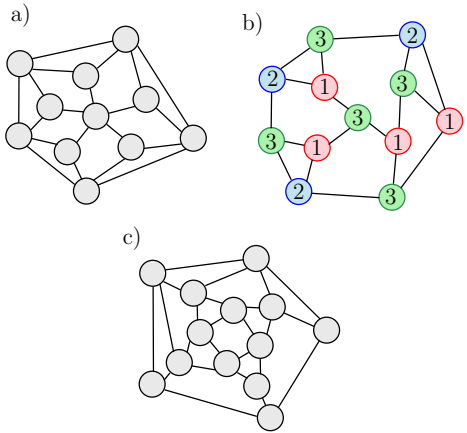

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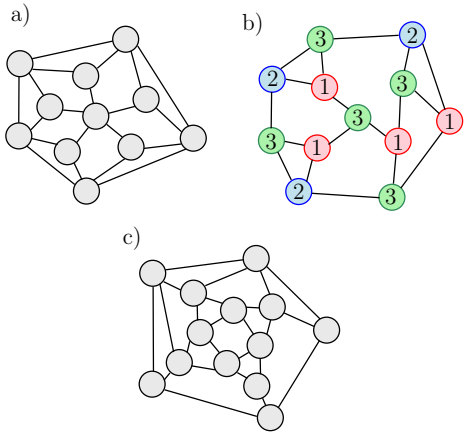
- Best known algorithm: $\approx 1.3289^n$ steps in the worst-case



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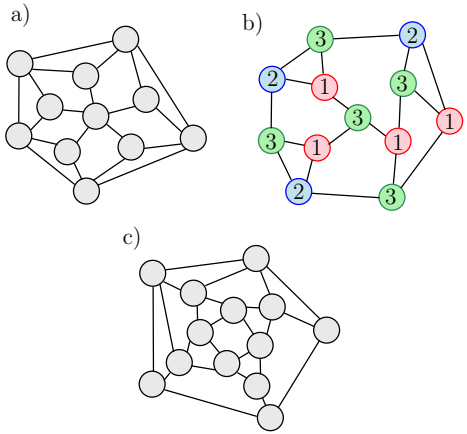
- Best known algorithm: $\approx 1.3289^n$ steps in the worst-case
- Theory: There exists $\varepsilon > 0$, so that no algorithm solves 3-coloring with less than $(1 + \varepsilon)^n$ steps in the worst-case



Problems and algorithms

Problems that have efficient algorithms:

Problems where exponential time is required:

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– Theory of computing: Finding efficient algorithms or proving that no efficient algorithms exist

Intended learning outcomes (ILOs)

- Design an efficient algorithm for 2-coloring a graph

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- Explain that according to theory of computing, there is no efficient algorithm for 3-coloring
- Explain that computational problems can be classified into categories based on whether efficient algorithms exist