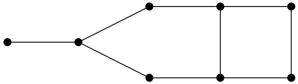
Induced Minors of Graphs

Tuukka Korhonen

Dagstuhl Seminar 25041: Solving Problems on Graphs: From Structure to Algorithms

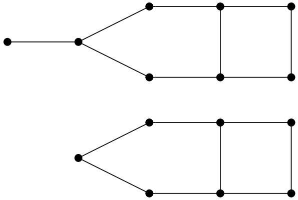
20 January 2025

Graph containment



Graph containment

1. Induced subgraph
   - vertex deletions



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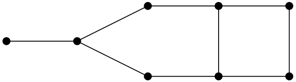

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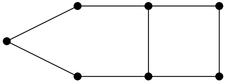

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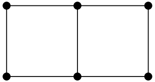

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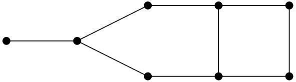

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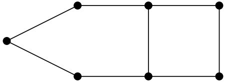

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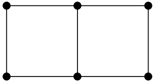

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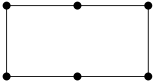

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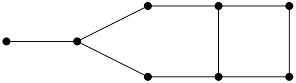

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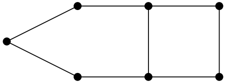

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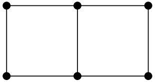

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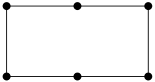

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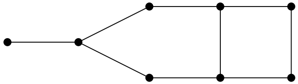

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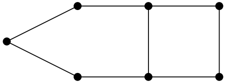

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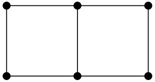

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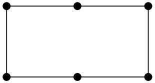

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(Contraction is defined so that it does not create self-loops or parallel edges)

Motivation

- Theory of induced minors $\Leftrightarrow$ dense generalization of the theory of graph minors?

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Which results from Graph Minors generalize to Induced Minors?

The Graph Minor Theorem

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Theorem (Robertson & Seymour, 1984-2004)

Let $\mathcal{C}$ be a minor-closed class of graphs. There exists a finite set of graphs $\mathcal{H}$, so that a graph G is in $\mathcal{C}$ if and only if G does not a graph from $\mathcal{H}$ as a minor.

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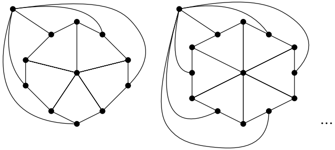
Theorem (Thomas, 1985)

This is not true for induced minors. (The induced minor order contains an infinite antichain.)



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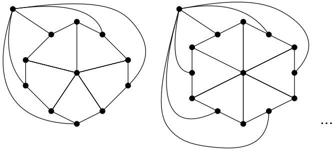

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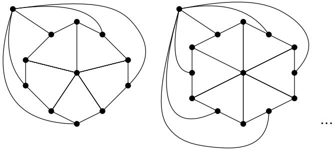

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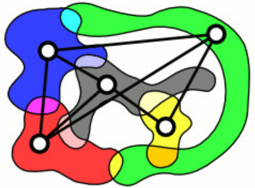
The class of string graphs is induced-minor-closed, but has an infinite number of minimal obstructions.



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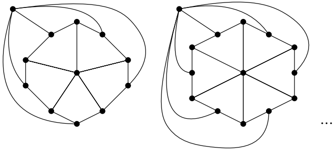

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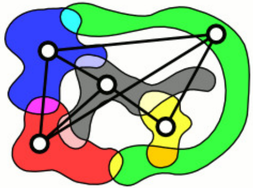

Many special classes well-quasi-ordered by induced minors [Thomas, 1985], [Ding, 1998], [Lozin & Mayhill, 2011], [Fellows, Hermelin & Rosamond, 2012], [Lewchalermvongs, 2015], [Błasiok, Kamiński, Raymond & Trunck, 2019]

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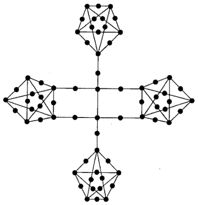

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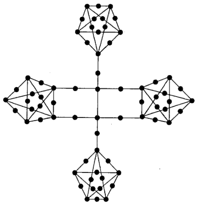

Theorem (K. & Lokshtanov, 2024)

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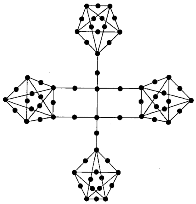

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Polynomial-time algorithms for special cases [Fellows, Kratochvíl, Middendorf & Pfeiffer, 1995], [Fiala, Kamiński, Paulusma, 2012], [van 't Hof, Kamiński, Paulusma, Szeider, Thilikos, 2012], [Golovach, Kratsch, Paulusma, 2013], [Belmonte, Golovach, Heggernes, van 't Hof, Kamiński, Paulusma, 2014], [Dallard, Dumas, Hilaire, Milanič, Perez, Trotignon, 2024], [Dallard, Dumas, Hilaire, Perez, 2025]

H-induced-minor-free graphs

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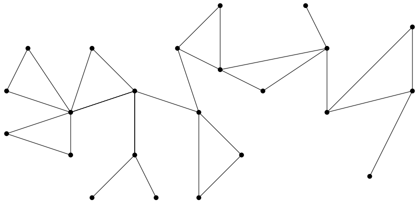

Example: C_4-minor-free graphs

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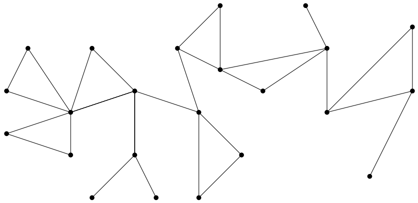

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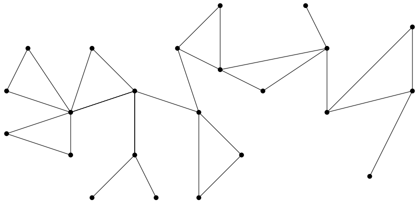

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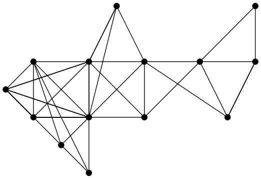

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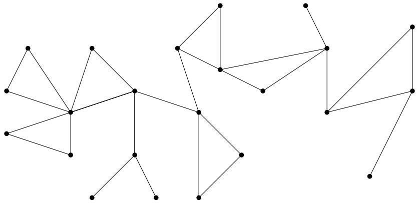

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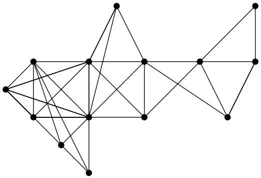

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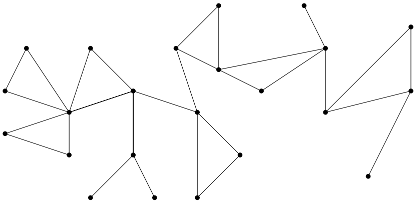

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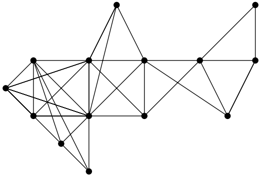

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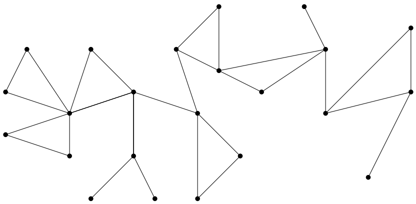

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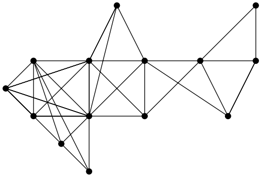

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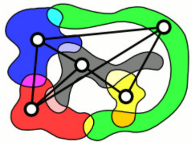

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$\Rightarrow$ Superclass of the string graphs

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   - Are H-induced-minor-free graphs similar to string graphs?

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   - Are H-induced-minor-free graphs similar to string graphs?
   - Subexponential-time algorithms/(quasi)Polynomial-time approximation schemes on H-induced-minor-free graphs?

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Are H-induced-minor-free graphs for planar H "tree-like"?

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If G excludes a planar graph H as an induced minor and has maximum degree Δ, then the treewidth of G is at most $\|H\|^{\mathcal{O}(1)} \cdot 2^{\Delta^5}$

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Note: Δ cannot be replaced by (much) stronger parameters! [Pohoata, 2014], [Sintiari & Trotignon, 2021], [Davies, 2022], [Bonamy, Bonnet, Déprés, Esperet, Geniet, Hilaire, Thomassé, Wesolek, 2024], [Abrishami, Alecu, Chudnovsky, Hajebi, Spirkl, 2024]

Excluding a planar graph: Algorithms

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- The Gartland-Lokshtanov structure conjecture would imply a Quasipolynomial-time approximation scheme.

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  - Claimed by [Lee, 2017] and [Bonnet, Hodor, K. & Masařík, 2023], but both have the same error

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Disprove: For every H there exists H', so that H-induced-minor-free graphs are intersection graphs of connected subgraphs of H'-minor-free graphs.

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- In other words, are dense models the only reason for hardness of H-induced-minor testing?

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