$\mathcal{O}(\sqrt{n})$-separators for minor-free graphs in linear-time

Édouard Bonnet, Tuukka Korhonen, Hung Le, Jason Li, Tomáš Masařík

The problem and result

Theorem (Alon, Seymour, Thomas, STOC 1990)

K_h-minor-free graphs have balanced separators of size $\mathcal{O}(h^{3/2} \cdot \sqrt{n})$.

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- $\mathcal{O}(n^{3/2})$ time algorithm from the proof
- $f(h) \cdot n^{1+o(1)}$ time for $\mathcal{O}(h \cdot \sqrt{n})$-separator [Kawarabayashi, Reed, FOCS 2010]
- $\mathcal{O}(\text{poly}(h) \cdot n^{5/4})$ time for $\mathcal{O}(h \cdot \sqrt{n \log n})$-separator [Wulff-Nilsen, FOCS 2011]
- $\mathcal{O}(\text{poly}(h) \cdot n)$ time for $\mathcal{O}(n^{1-\varepsilon})$-separator [Wulff-Nilsen, FOCS 2011]

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Deterministic for finding separator or concluding the existence of K_h-minor, randomized for finding the minor model.

Techniques

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Techniques:

- Low-diameter decomposition of K_t-minor-free graphs of [Klein, Plotkin, Rao]
- Novel? technique of constructing routing trees via BFS combined with weight-updates