Induced-Minor-Free Graphs: Separator Theorem, Subexponential Algorithms, and Improved Hardness of Recognition

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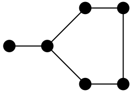
UNIVERSITY OF BERGEN

based on joint work [SODA'24] with Daniel Lokshtanov from UCSB

University of Warsaw algorithms seminar

8 December 2023

Graph containment



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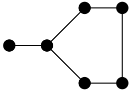
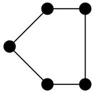


1. Induced subgraph
   - vertex deletions



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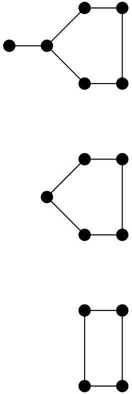
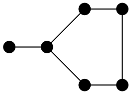


1. Induced subgraph
   - vertex deletions
2. Induced minor
   - vertex deletions
   - edge contractions

Graph containment



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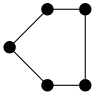

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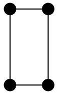

3. Minor
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   - edge contractions
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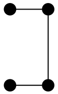

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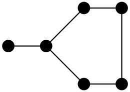

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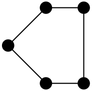

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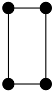

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In this talk, all graphs are simple!

Graph classses defined by containment

- For a graph H, can define graph classes by excluding H
  - H-minor-free graphs
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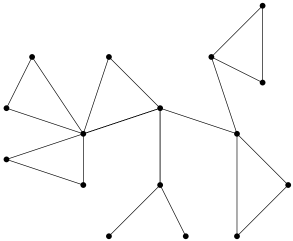
- Example: C_4-minor-free graphs



- Every biconnected component is a triangle
- Chordal and treewidth ≤ 2

Induced-minor-free graphs

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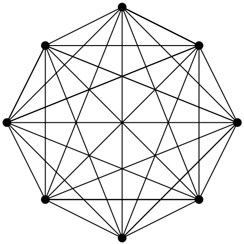
- Complete graph contains only complete graphs as induced minors



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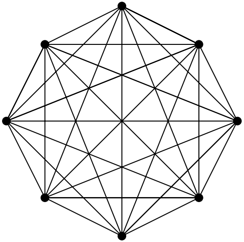
$\Rightarrow$ C_4-induced-minor-free graphs have unbounded treewidth



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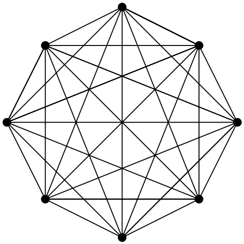
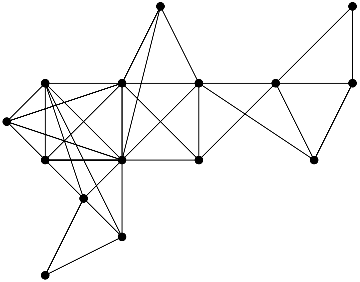
- C_4-induced-minor-free graphs $\Leftrightarrow$ chordal graphs





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- (In my opinion) the main open problem:

Open problem (Dallard, Milanic, Storgel '21)

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Solved for:

- $H = P_k$ [Gartland and Lokshtanov FOCS'20]
- $H = C_k$ [Gartland, Lokshtanov, Pilipczuk, Pilipczuk, Rzazewski STOC'21]
- $H = W_4$, $H = K_5^-$, and $H = K_{2,q}$ [Dallard, Milanic, Storgel '21]
- $H = tC_3$ [Bonamy, Bonnet, Depres, Esperet, Geniet, Hilaire, Thomasse, Wesolek SODA'23]
- $H = K_1 + tK_2$ and $H = tC_3 \uplus C_4$ [Bonnet, Duron, Geniet, Thomasse, Wesolek ESA'23]
- Bounded-degree input graphs [K. JCTB'23]

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Open problem (Lokshtanov)

Does this capture all H-induced-minor-free graphs?

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- Generalizes separator theorems on string graphs: $\mathcal{O}(m^{3/4}\sqrt{\log m})$ by [Fox and Pach'10], $\mathcal{O}(\sqrt{m}\log m)$ by [Matousek '14], $\mathcal{O}(\sqrt{m})$ by [Lee'17]

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- The theorem of [Lee'17] works also on intersection graphs of connected subgraphs of minor-free graphs

Algorithmic applications

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- Independent set, feedback vertex set, $\mathcal{F}$-minor-deletion...

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Set $t = |H|^4 \cdot \sqrt{m}$: If separator, we are done.

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- Issue: No control over non-indepedent edges
- Solution: Subdivide H three times, get a bad model of subdivided graph, reroute to a good model of H

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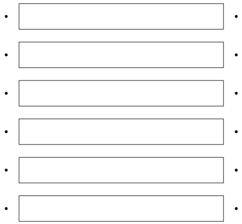
- Furthermore, H is a tree and the proof also gives NP-hardness
- Solves two open problems of [Fellows, Kratochvil, Middendorf, & Pfeiffer '95], who asked the existence of such H that is (1) planar (2) tree

Proof idea

- Main intermediate problem: Multicolored 6-disjoint induced paths, with special structure:



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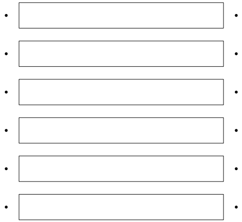

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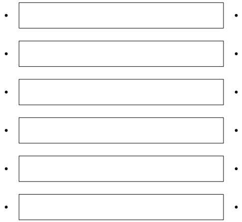

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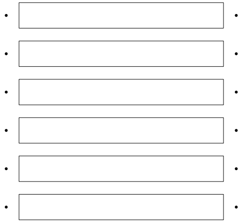
Step 2: Reduction to rooted induced minors, and then by using pathwidth to induced minors



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2. Are all H-induced-minor-free graphs intersection graphs of connected subgraphs of H'-minor-free graphs?

Conclusion

1. $\mathcal{O}(\sqrt{m})$-separator theorem
2. $2^{o(n/\log^3 n)}$ hardness of H-induced-minor-containment for fixed tree H

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Thanks!