Fixed-Parameter Tractability of Maximum Colored Path and Beyond

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SODA 2023

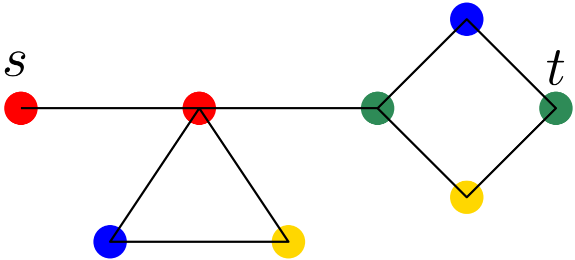
24 January 2023

Maximum Colored Path

MAXIMUM COLORED s, t-PATH

Input: Vertex-colored undirected graph, vertices s and t, and an integer k.

Task: Find an s, t-path containing at least k different colors.



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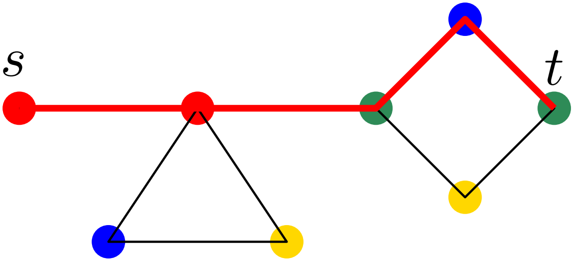

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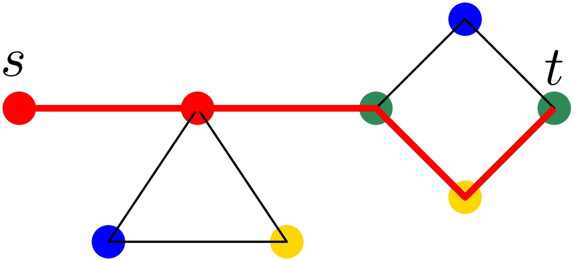

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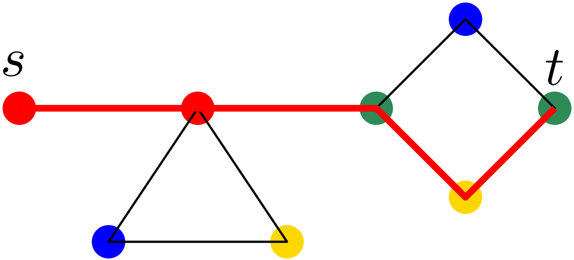

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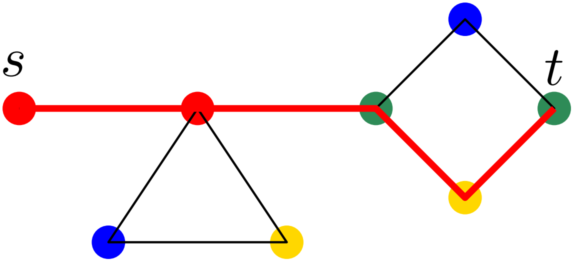

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- *Path* does not contain repeated vertices
- Color may repeat multiple times in the path, and it can contain more than k colors

Fixed-parameter tractability of Maximum Colored Path

MAXIMUM COLORED s, t-PATH

Input: Vertex-colored undirected graph, vertices s and t, and an integer k.

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Theorem

There is a $2^k n^{\mathcal{O}(1)}$ time randomized algorithm for maximum colored s, t-path. Moreover, the algorithm returns the shortest solution.

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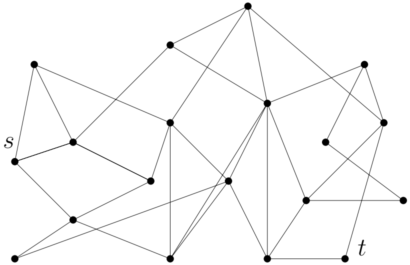
- Assuming set cover conjecture, no $(2 - \varepsilon)^k n^{\mathcal{O}(1)}$ time algorithm for any $\varepsilon > 0$
- NP-hard for directed graphs already when $k = 2$

Application: Longest s, t-Path

LONGEST s, t-PATH

Input: Undirected graph, vertices s and t, and an integer k.

Task: Find an s, t-path of length at least k.



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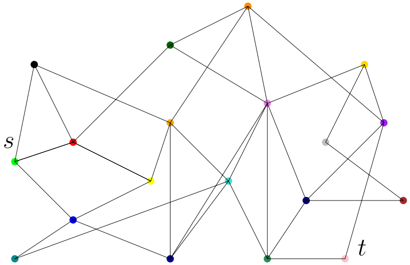

- Longest s, t-path reduces to maximum colored s, t-path by coloring all vertices with different colors

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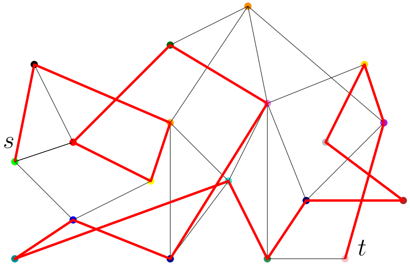

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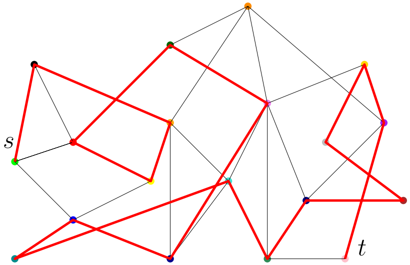

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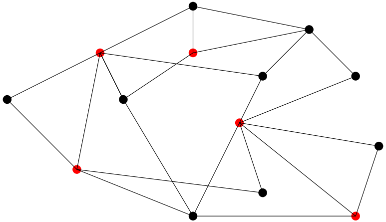
- Previous best algorithm $4^k n^{\mathcal{O}(1)}$ time [Fomin, Lokshtanov, Panolan, Saurabh, Zehavi'18]

Application: T-cycle

T-CYCLE

Input: Undirected graph and a set of terminal vertices T.

Task: Find a cycle that visits each vertex in T.



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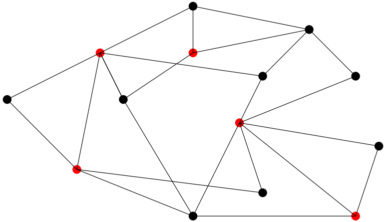

- $2^{|T|} n^{\mathcal{O}(1)}$ time algorithm for T-cycle [Björklund, Husfeldt, Taslaman, SODA'12]

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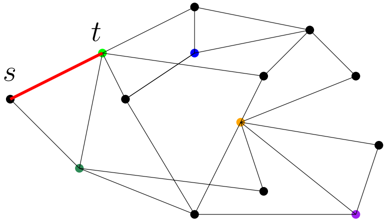

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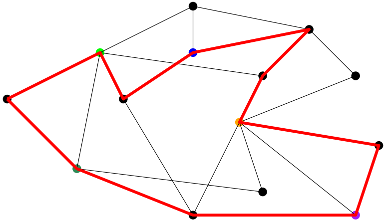

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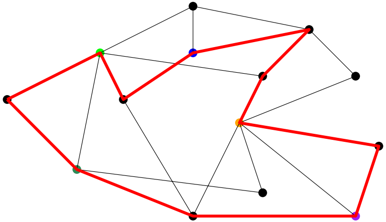

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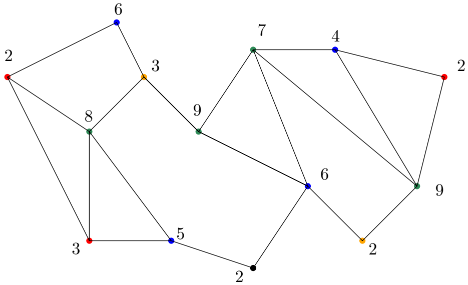
$\Rightarrow$ $2^{|T|} n^{\mathcal{O}(1)}$ time algorithm also via our result

- Allows for generalizations, e.g., need to visit k vertices of large T

Beyond Maximum Colored Path

Input:

- Colored positive-integer weighted undirected graph



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Input:

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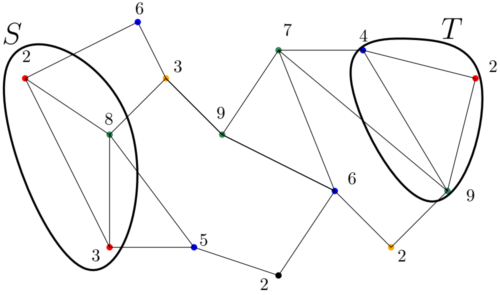
- Two sets of vertices S and T



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Input:

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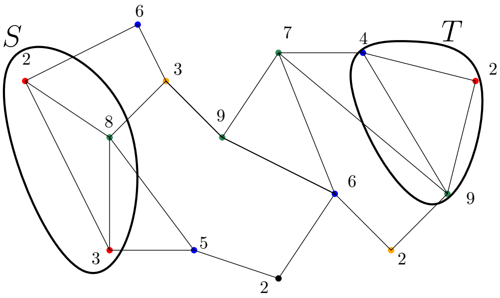

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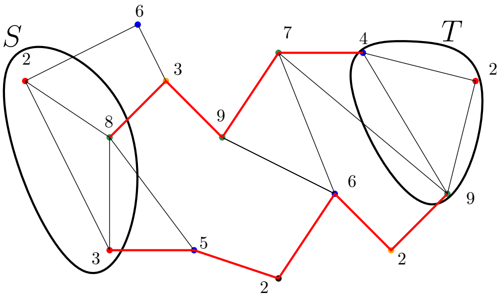
Problem:

- Find p vertex-disjoint paths starting in S and ending in T so that



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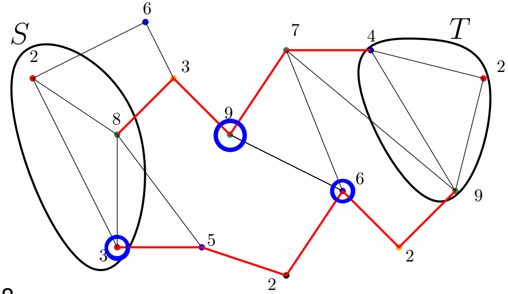

Here, $p = 2$, $k = 3$, and $w = 18$.

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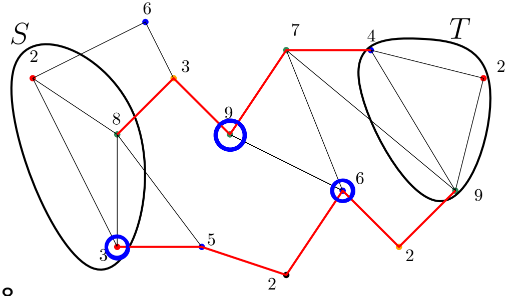

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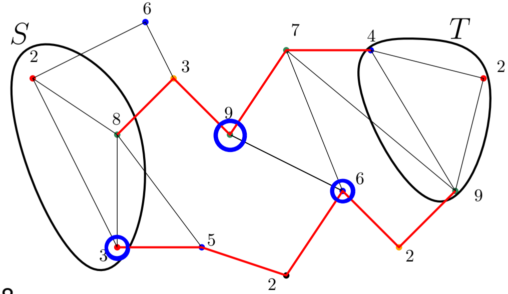

Here, $p = 2$, $k = 3$, and $w = 18$.

Main theorem: Randomized algorithm with time complexity $2^{k+p} n^{\mathcal{O}(1)} w$.

The Algorithm

In this talk, we focus on:

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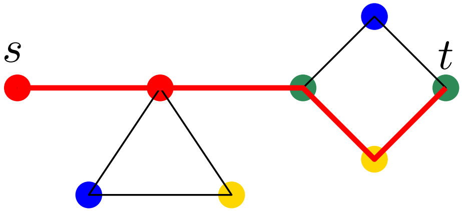

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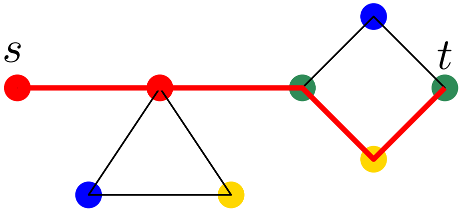

- Based on algebraic approach, extending ideas that were developed by [Björklund, Husfeldt, Taslaman'12], for T-cycle [Björklund'14] for Hamiltonicity, and [Björklund, Husfeldt, Kaski, Koivisto'17] for k-path

Algebraic approach

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- By DeMillo–Lipton–Schwartz–Zippel lemma, the problem is then solved in $2^k n^{\mathcal{O}(1)}$ time by evaluating $p(x_1, \ldots, x_m)$ for random $x_1, \ldots, x_m$.

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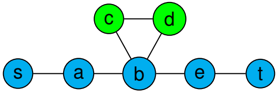
- Characteristic 2?
  - $x + x = 0$ for any x

Polynomial over labeled walks



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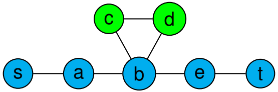

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- For each edge uv associate variable $f_e(uv)$
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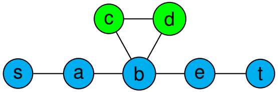

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For a labeled walk W, associate monomial $f(W)$ that is product of edge variables, vertex variables of labeled vertices, and color-label pair variables corresponding to labeled vertices

$$f(sa\overset{1}{b}c\overset{2}{d}bet) = f_e(sa)f_e(ab)f_e(bc)f_e(cd)f_e(db)f_e(be)f_e(et)f_v(b)f_v(d)f_c(\textcolor{blue}{\bullet}, 1)f_c(\textcolor{green}{\bullet}, 2)$$

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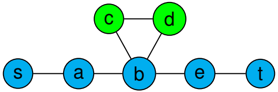

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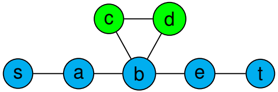

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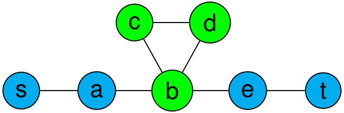
- Define $f(\mathcal{C}_\ell) = \sum_{W \in \mathcal{C}_\ell} f(W)$

Yes-instances



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(1) Exists a k-colored (s, t)-path of length ℓ $\Rightarrow$ $f(\mathcal{C}_\ell)$ non-zero

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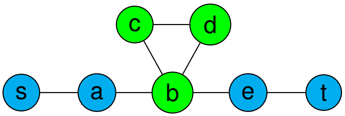

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- Labeled walk $s\overset{1}{a}\overset{2}{b}et \in \mathcal{C}_5$ contributes the monomial $f_e(sa)f_e(ab)f_e(be)f_e(et)f_v(a)f_v(b)f_c(\bullet, 1)f_c(\bullet, 2)$

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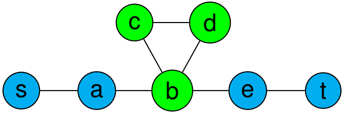

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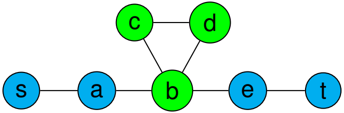

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Combination of arguments: 18 cases and 14 pages

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- Open problem: Is there a $1.99^k n^{\mathcal{O}(1)}$ time algorithm for longest (s, t)-path?

Thank you!