# PACE solver description: SMS

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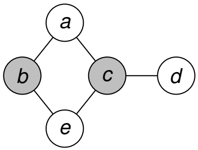
# Branching on Minimal Separators

- $\Delta(G)$ – minimal separators of graph $G$

**Theorem [Deogun, Kloks, Kratsch, and Müller, 1999]**

If $G$ is not complete then

$$\mathtt{td}(G) = \min_{S \in \Delta(G)} \left( |S| + \max_{C \in \mathcal{C}(G \setminus S)} \mathtt{td}(G[C]) \right)$$



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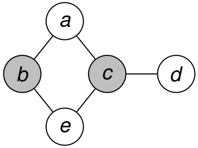
- Branching algorithm, where each subproblem of type $\mathtt{td}(G[X]) \leq_? k$ for induced subgraph $G[X]$ and number $k$



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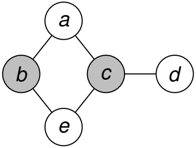
- Branching algorithm, where each subproblem of type $\mathtt{td}(G[X]) \leq_? k$ for induced subgraph $G[X]$ and number $k$
- Sufficient to consider $\Delta_k(G) = \{|S| \leq k \mid S \in \Delta(G)\}$



# Implementation overview

- Before search
  - Preprocessing techniques
    - Kernelization by vertex cover [Kobayashi and Tamaki, 2016]
    - Tree subgraph elimination
  - Upper bound algorithm
    - Sample minimal triangulations with minfill heuristic

- During search
  - Lower bound techniques
    - Long paths, cycles, and clique minors [Bodlaender and Koster, 2011]
    - Data structure to find bounds on $\mathtt{td}(G[Y])$ with $Y \subset X$ given $X$
    - Graph isomorphism hashtable
  - Enumerating $\Delta_k(G)$ (small minimal separators)

# Enumerating $\Delta_k(G)$

- First use heuristic algorithm, but rerun with exact
  1. Heuristic
     - Simple modification of algorithm of [Berry, Bordat, and Cogis, 2000]
     - Outputs a set $\Delta_h \subseteq \Delta_k(G)$ in $O^*(|\Delta_h|)$ time
     - **Incorrect** but works well in practice
  2. Exact
     - Algorithm of [Tamaki, 2019]
     - $\approx 2 - 10$ times slower than the heuristic
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- We can generate $\Delta_{k-|S|}(G[C])$ from $\Delta_k(G)$
  - ▸ We do this when $|C| \geq |V(G)|/2$

# Conclusion

- Approach based on branching from small minimal separators
- Enumeration of small minimal separators the runtime bottleneck

## Open problem

Is there algorithm that outputs $\Delta_k(G)$ in $O^*(|\Delta_k(G)|)$ time?

# The end

Thank you for your attention!

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