FPT Algorithms for Treewidth via Local Improvement

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UNIVERSITY OF BERGEN

based on joint work with
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[1] University of California Santa Barbara

Graph Decompositions: Small Width, Big Challenges

27 October 2022

New Results on Computing Treewidth

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There is a $k^{\mathcal{O}(k/\varepsilon)}n^4$ time $(1+\varepsilon)$-approximation algorithm for treewidth.

Outline

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1. How to improve a tree decomposition

Suffices to solve the *Subset treewidth* problem

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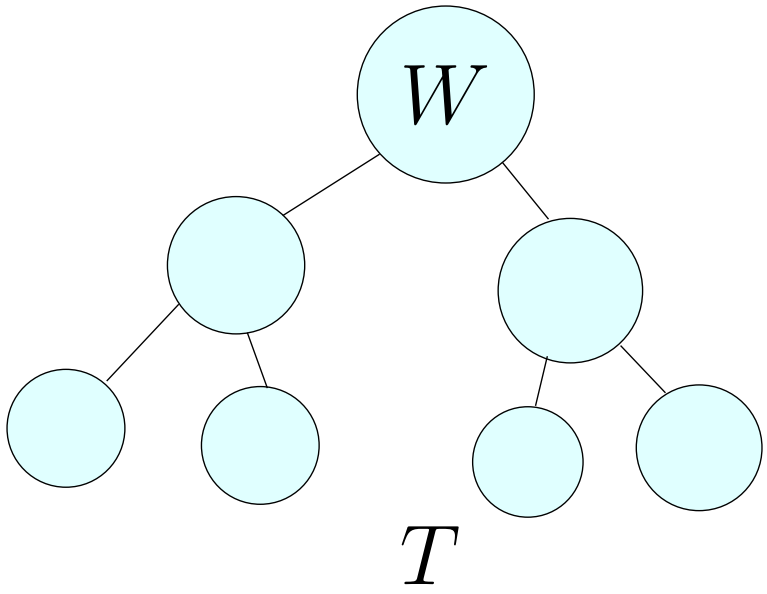
2. Solving the subset treewidth problem

Algorithms for subset treewidth that then imply algorithms for treewidth

How to improve a tree decomposition

Setting

Suppose we have a tree decomposition T whose largest bag is W



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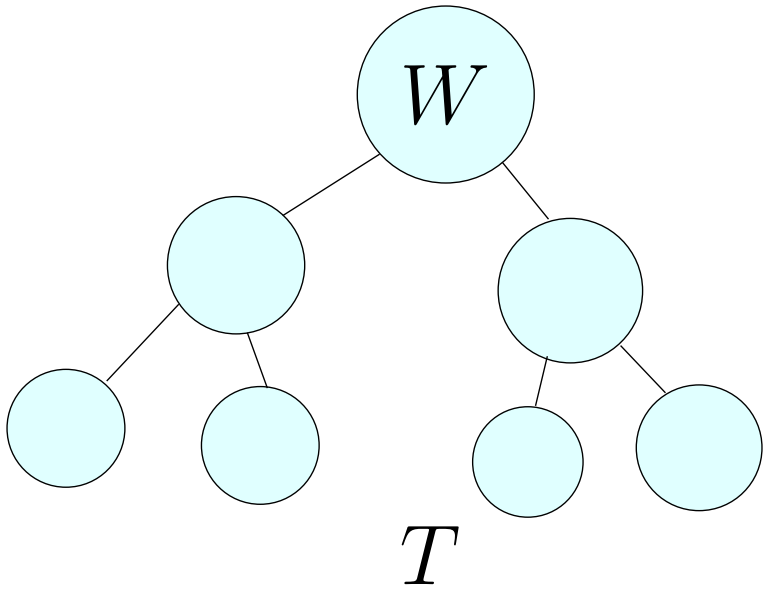

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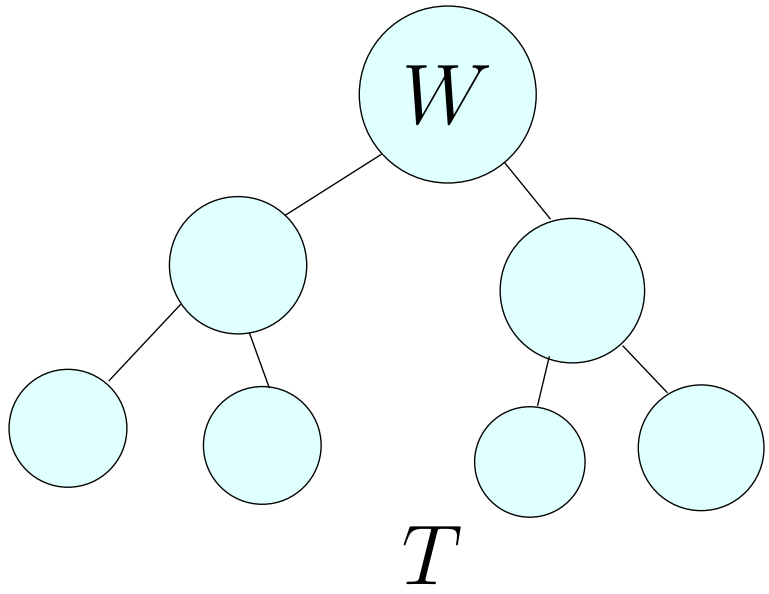

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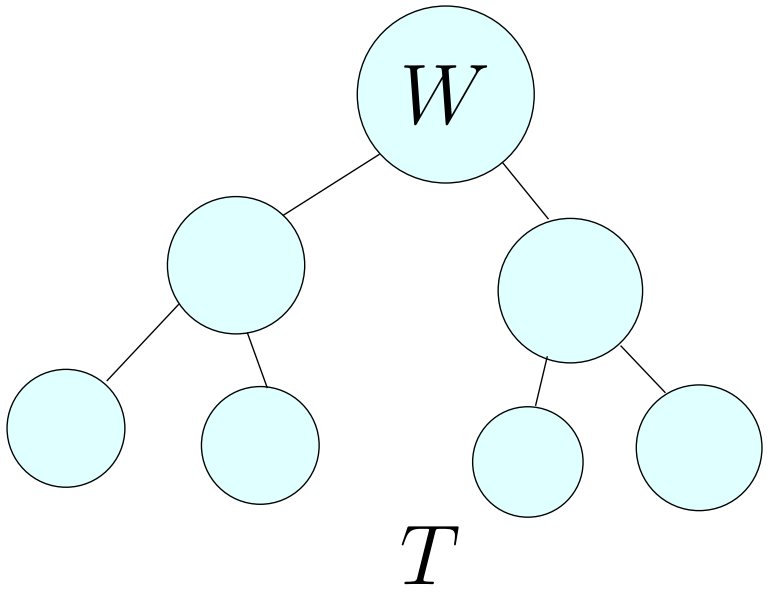

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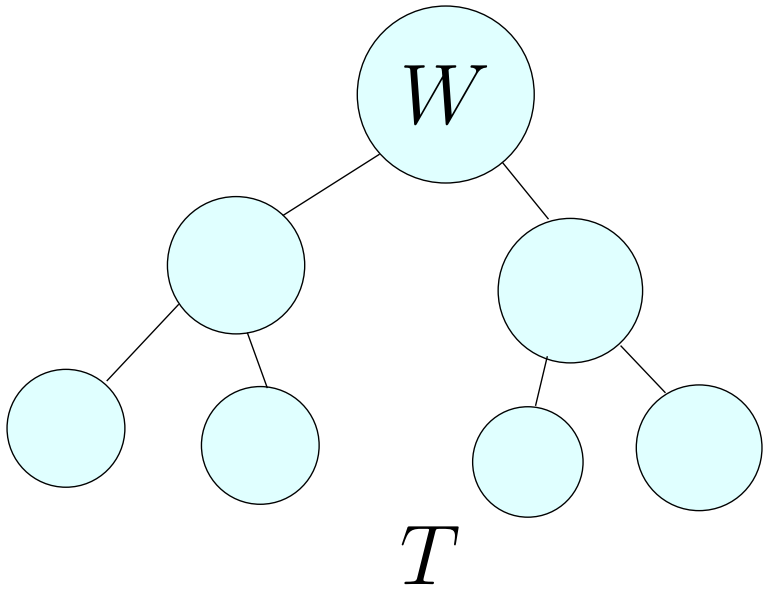
Repeat for $\mathcal{O}(\text{tw}(G) \cdot n)$ iterations to get an (approximately) optimal tree decomposition



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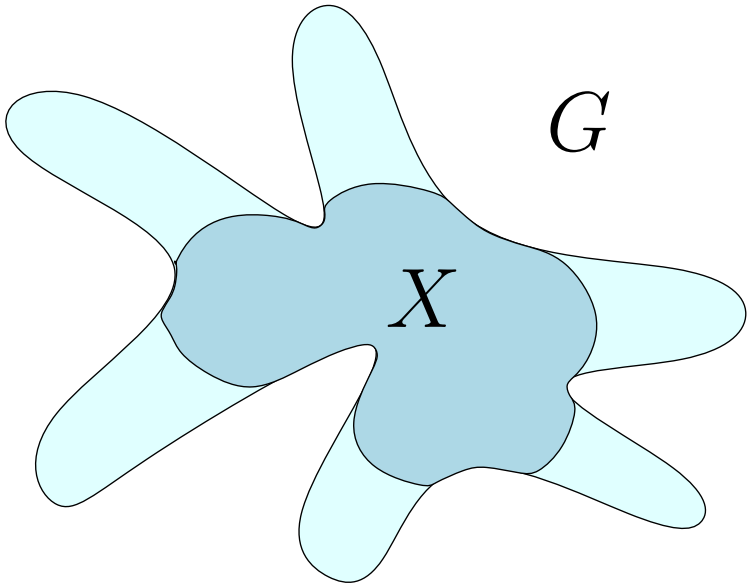

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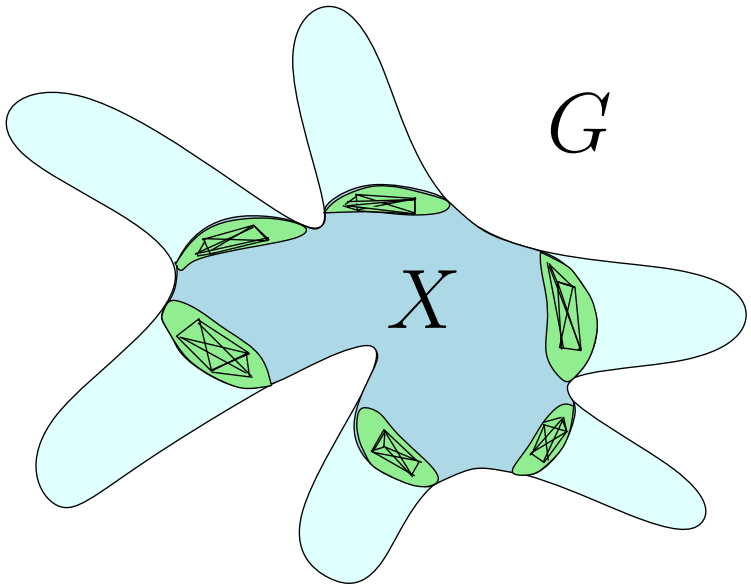

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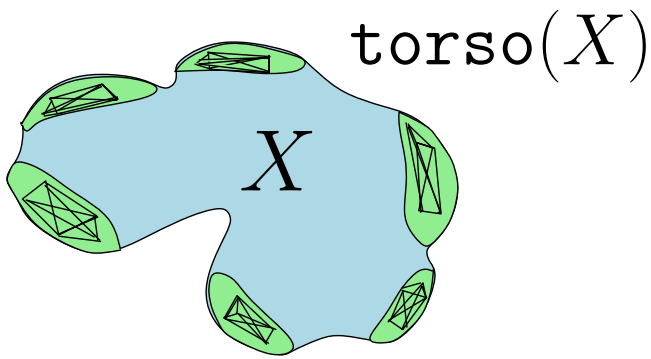

- Make neighborhoods of components of $G \setminus X$ into cliques
- Delete $V(G) \setminus X$

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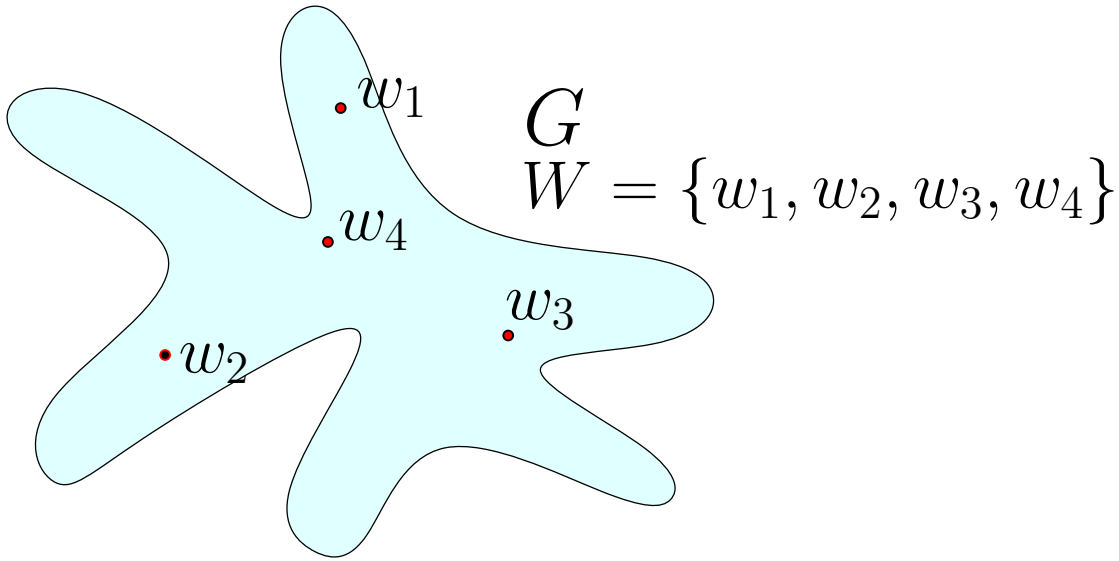

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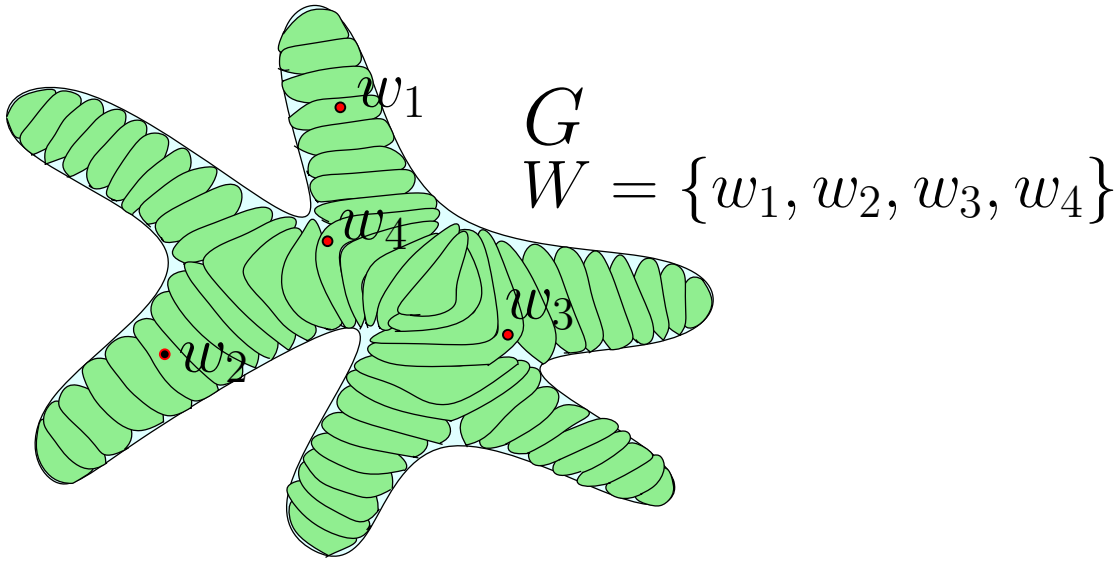

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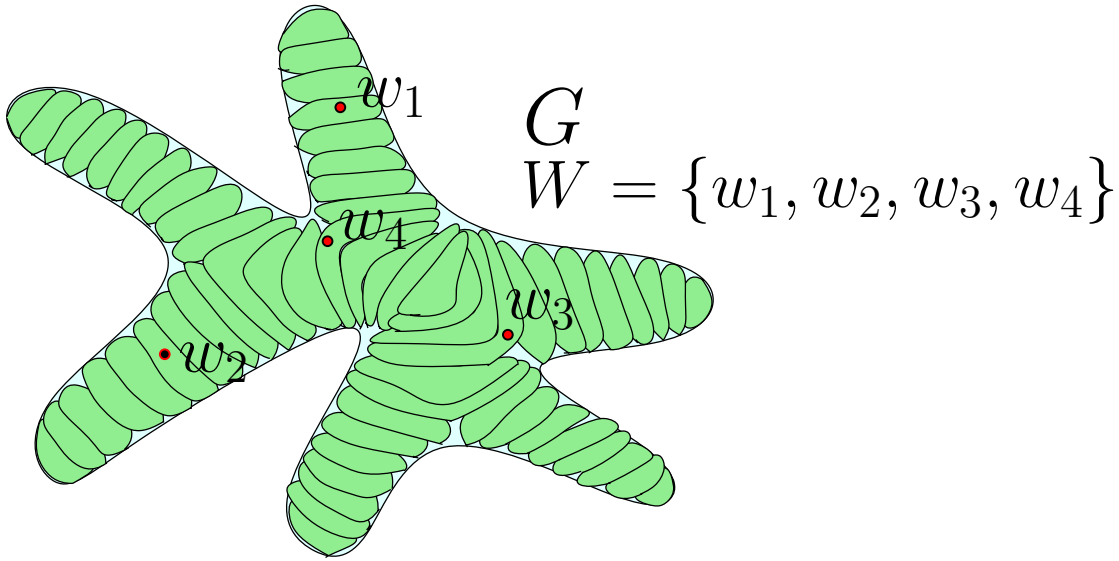

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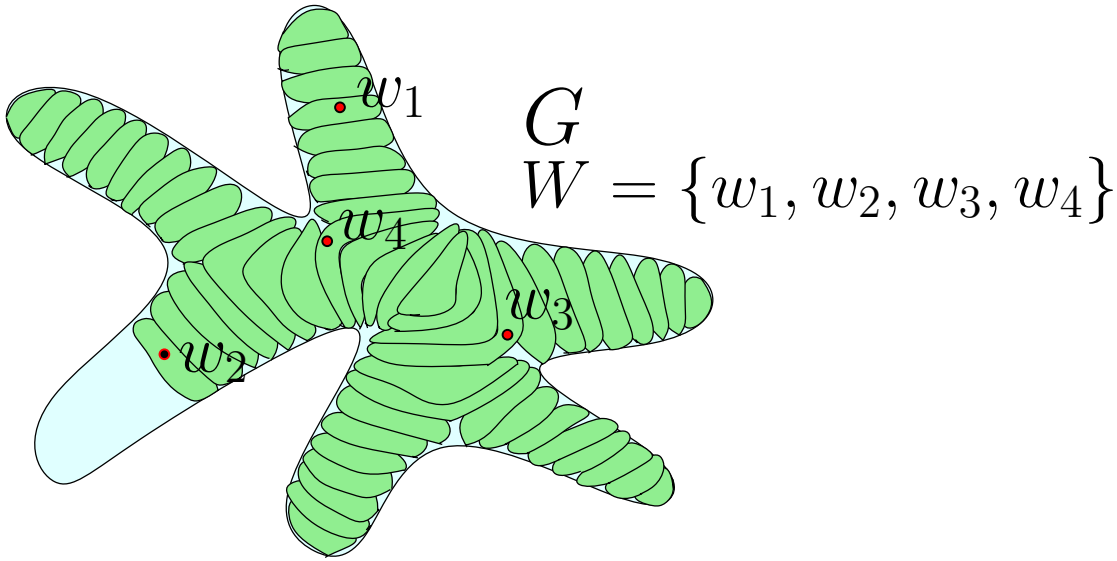

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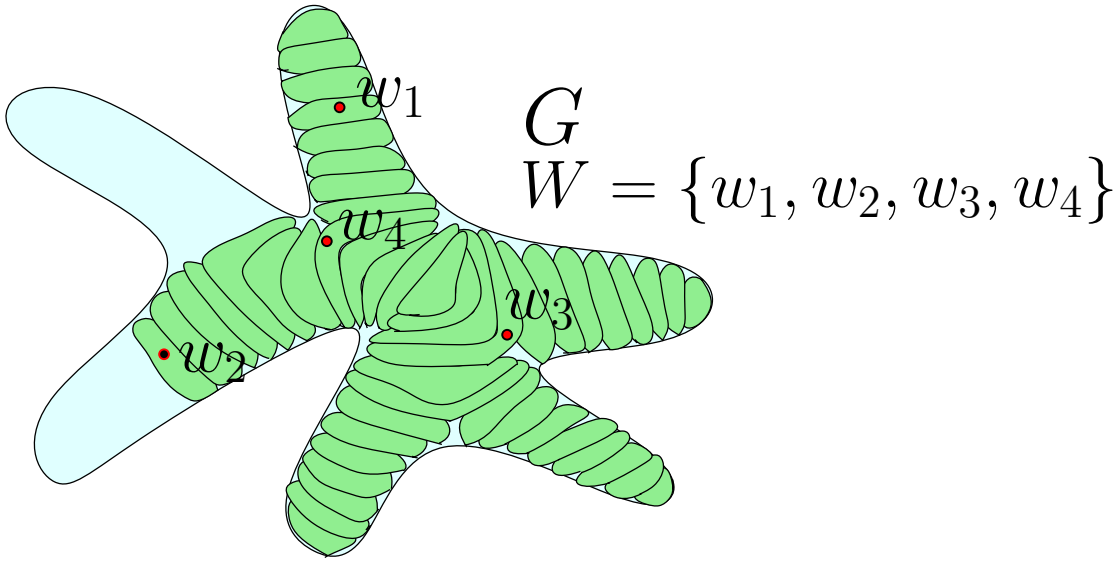

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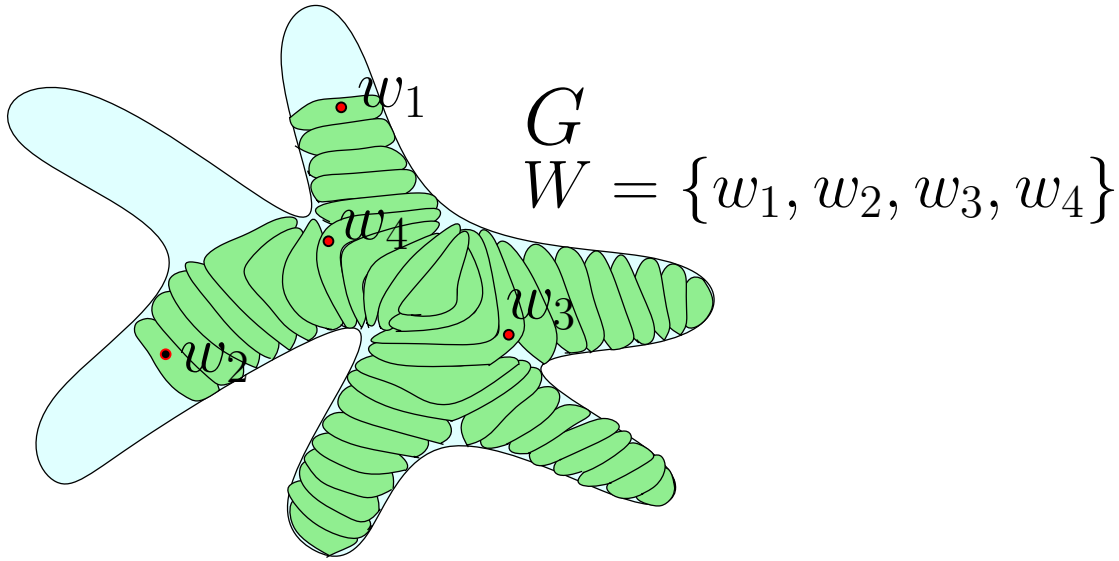

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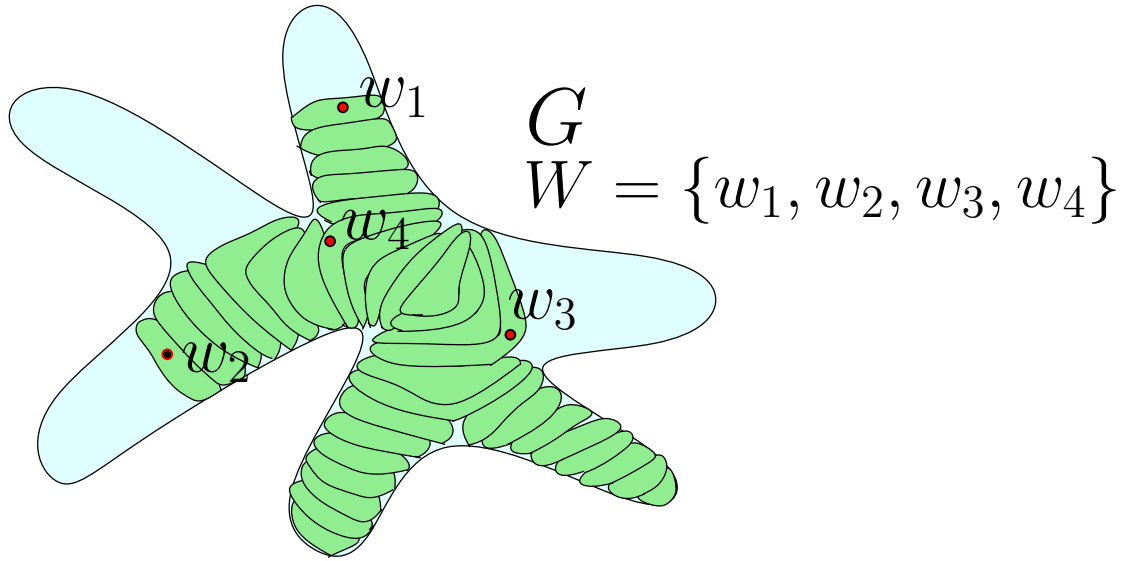

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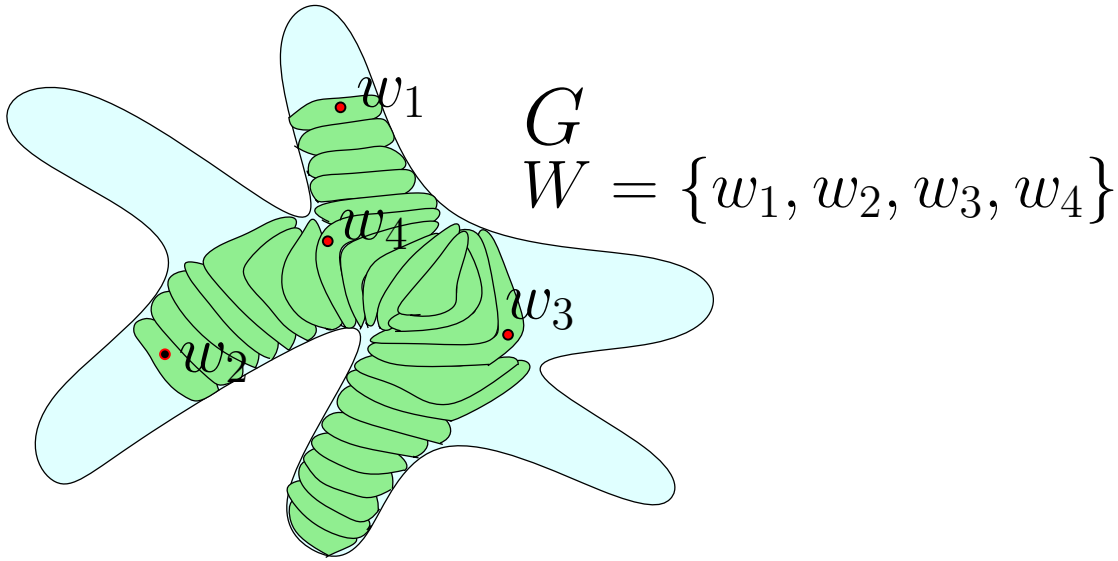

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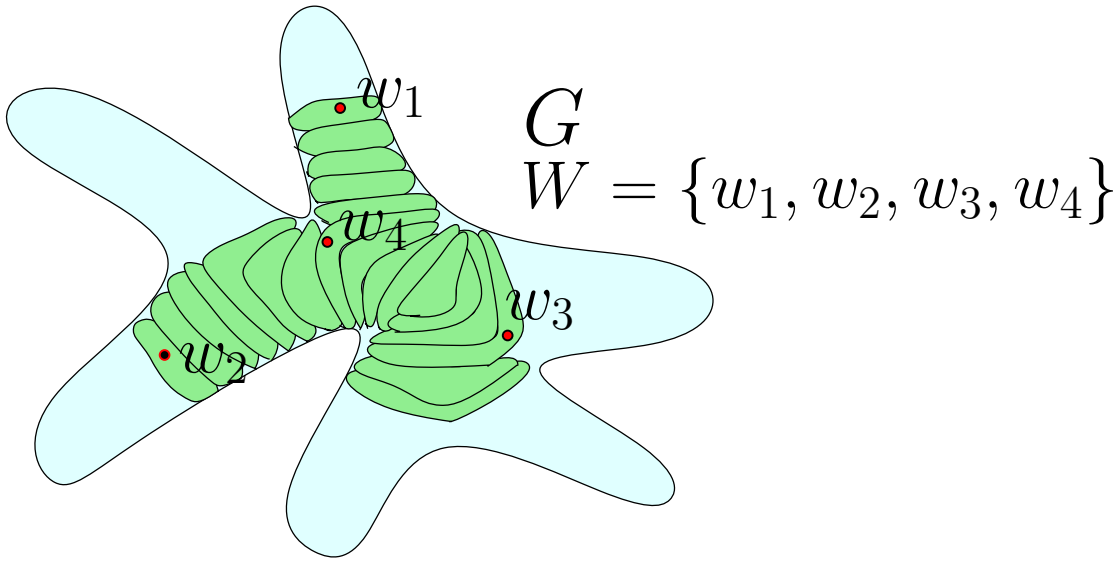

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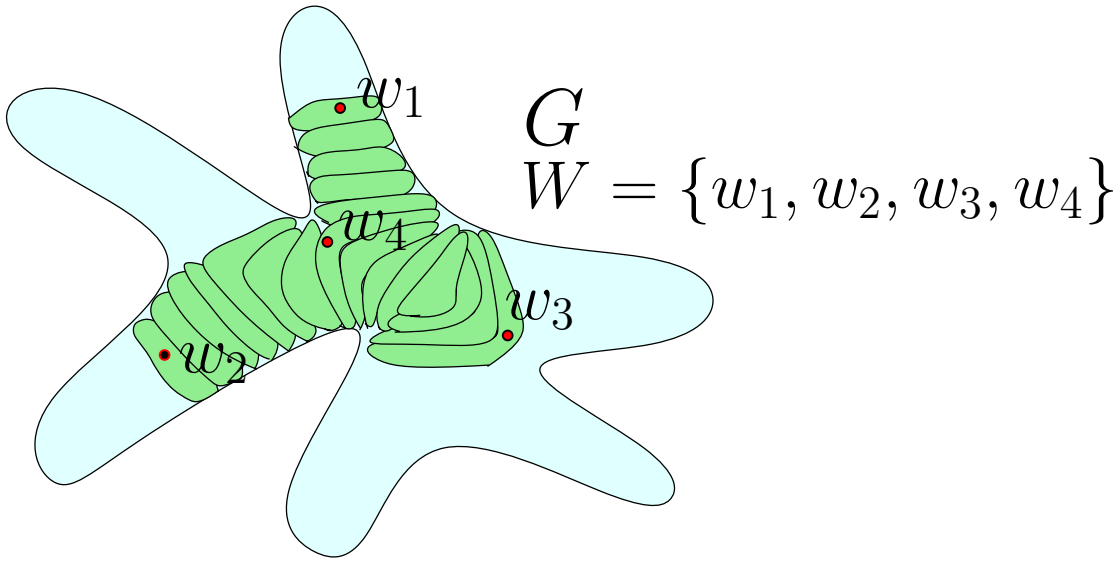

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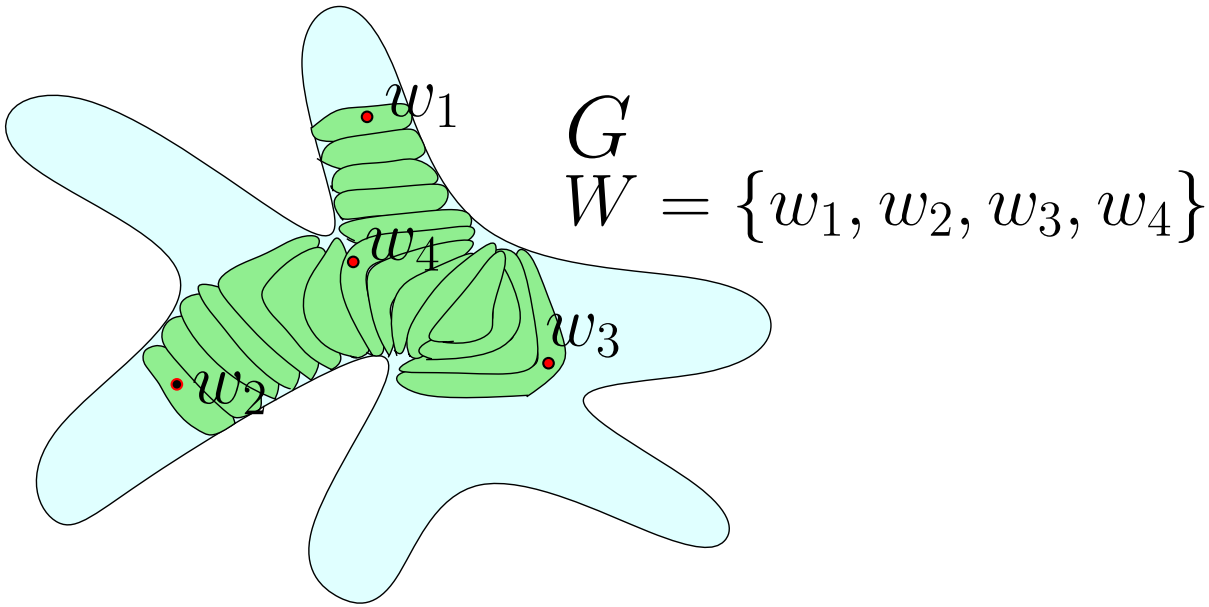
Big-leaf formulation:



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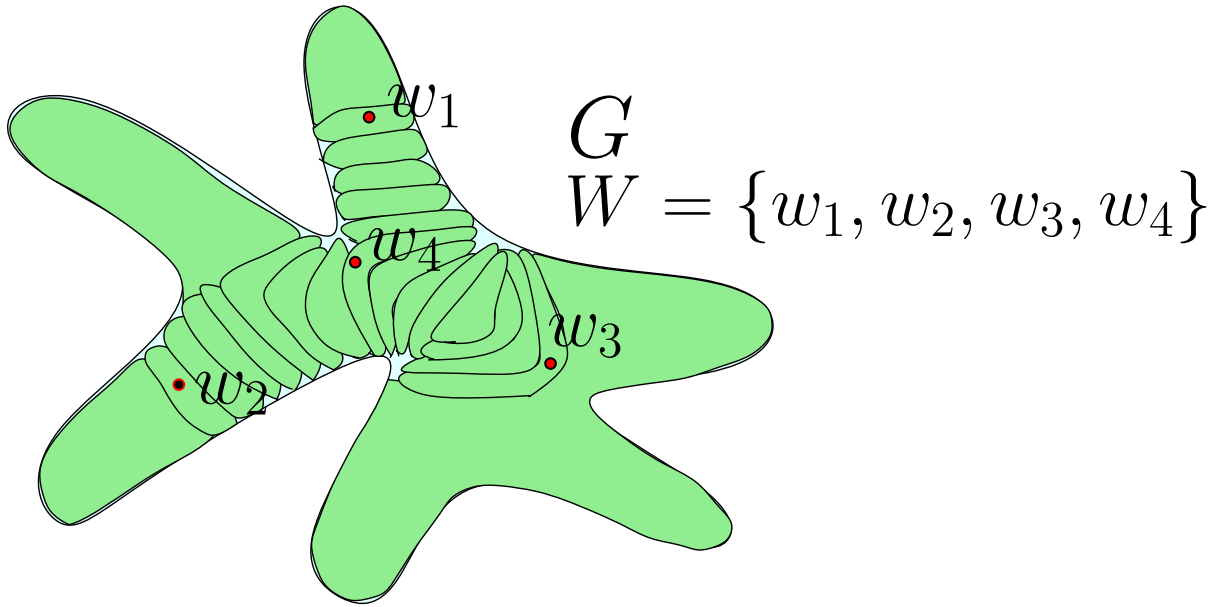
Big-leaf formulation:

- Find a tree decomposition of G whose internal bags have size $\leq |W| - 1$ and cover W, but leaf bags can be arbitrarily large



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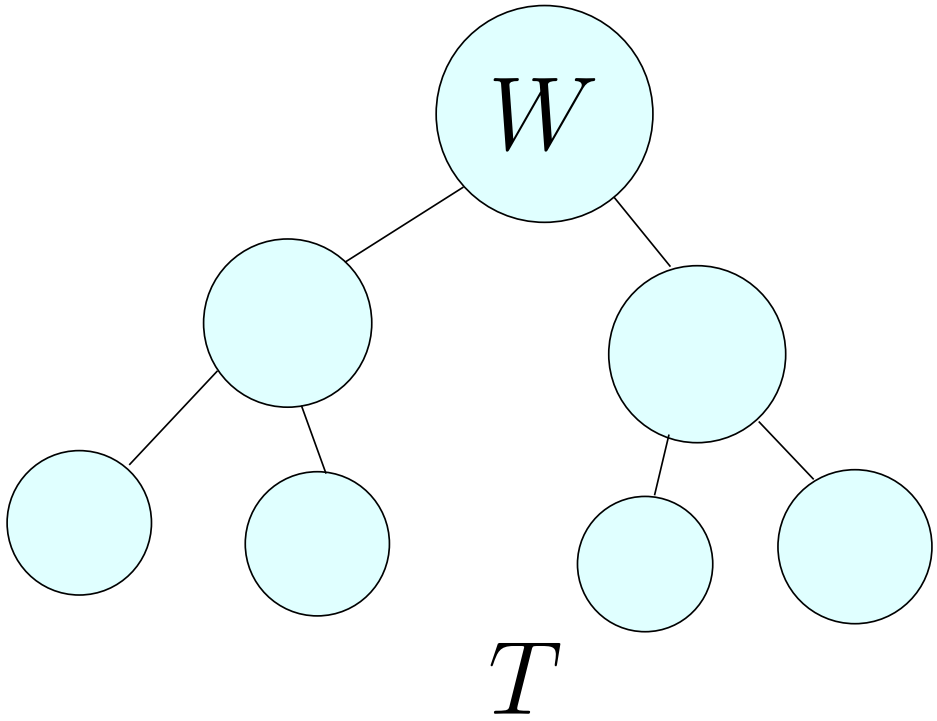

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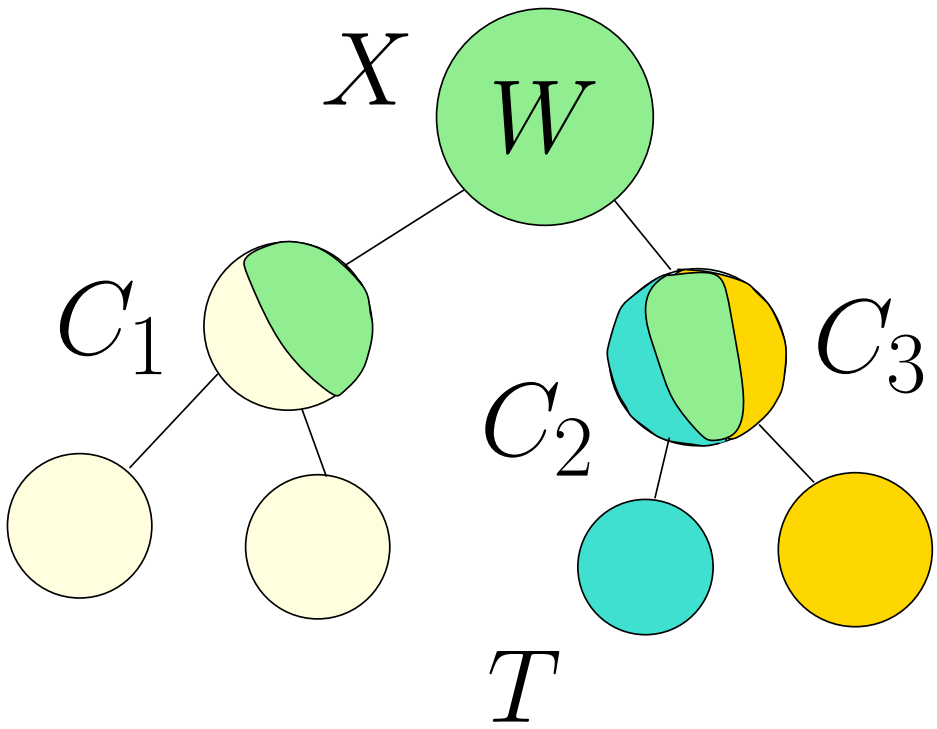

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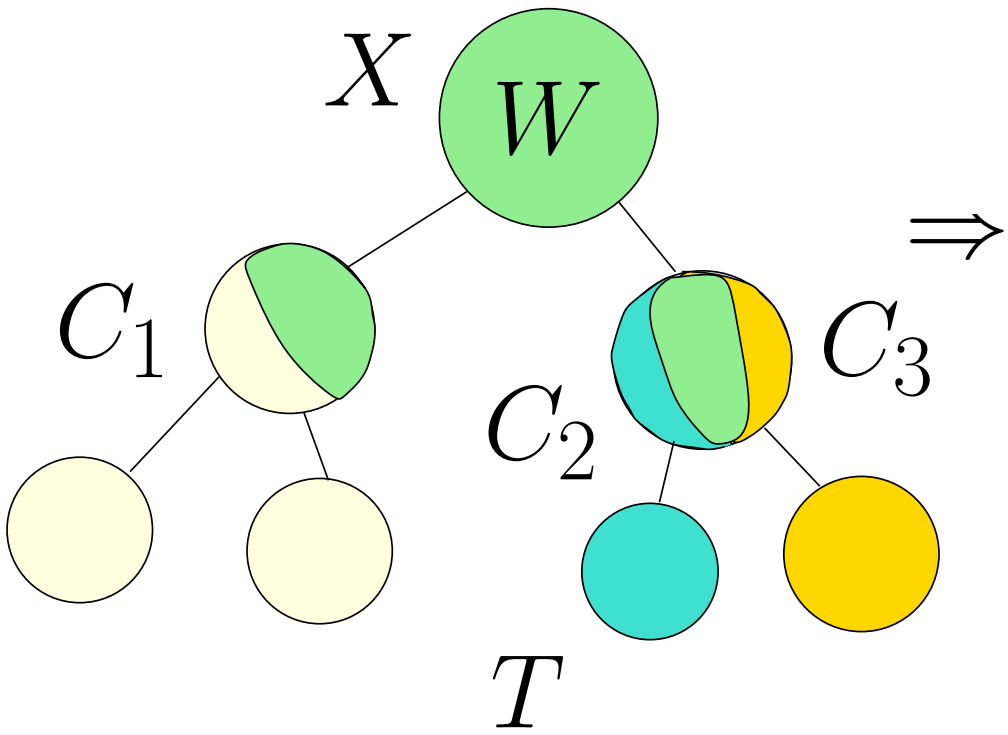

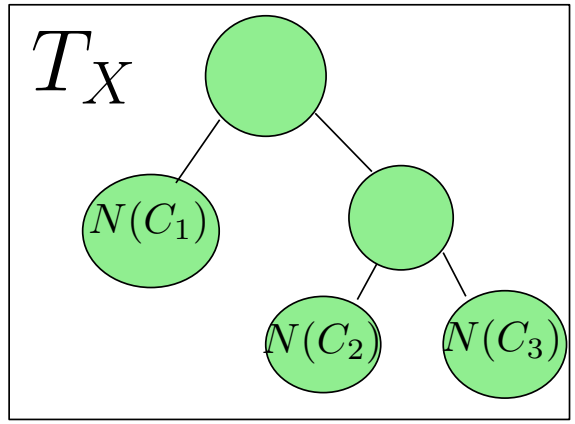

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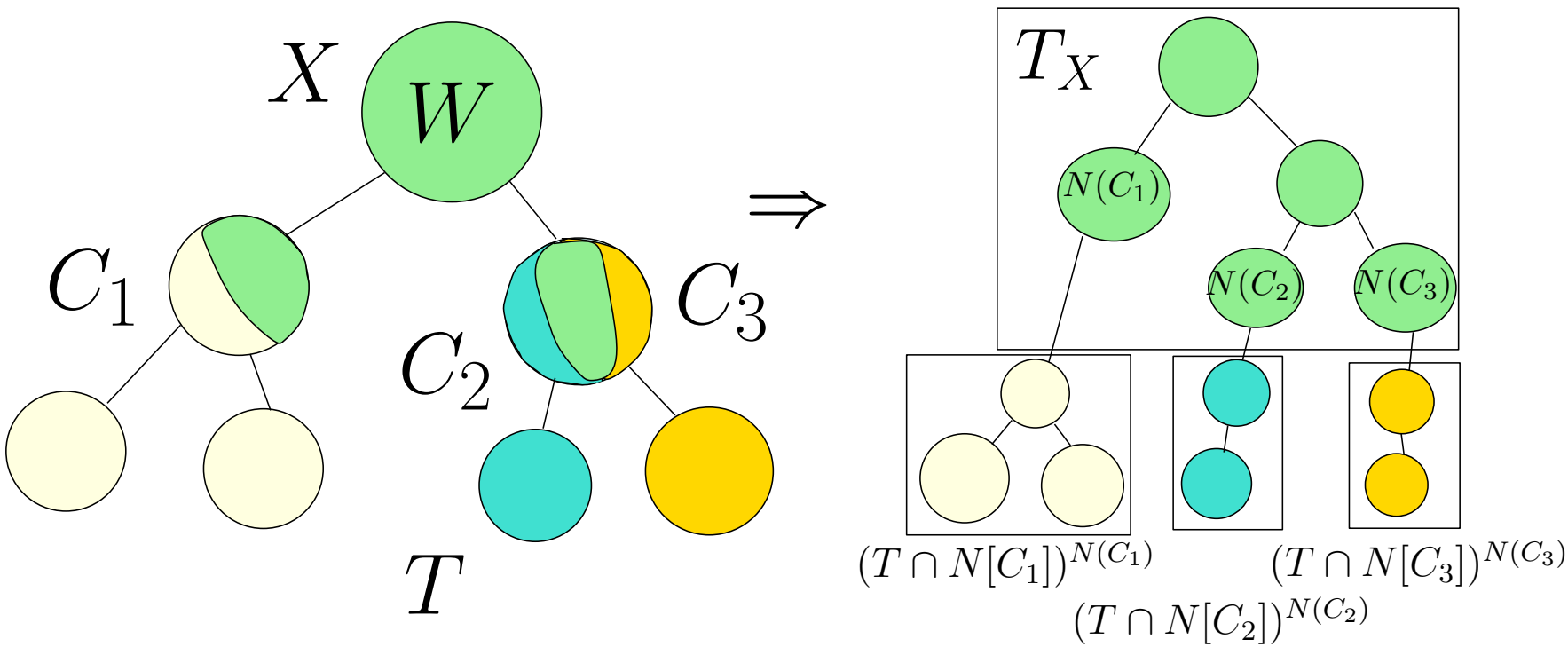
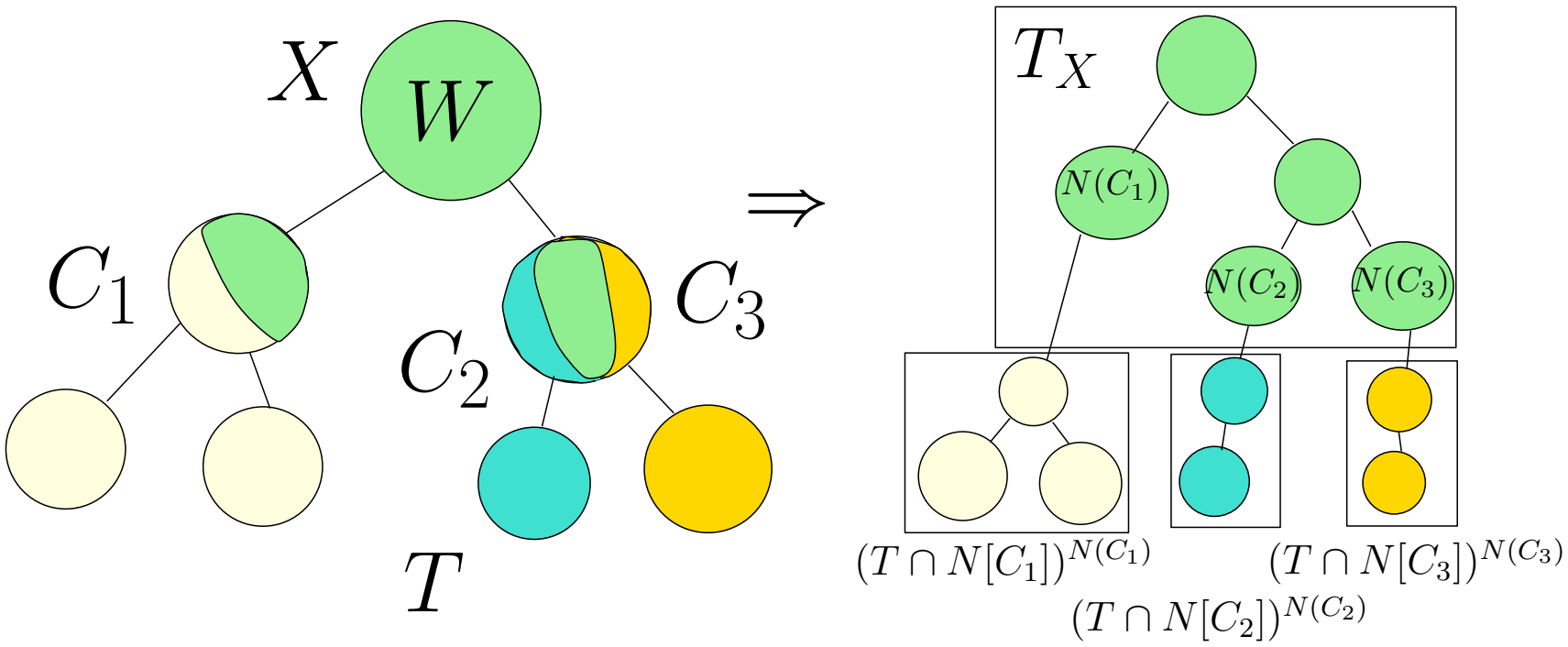


Does T improve?



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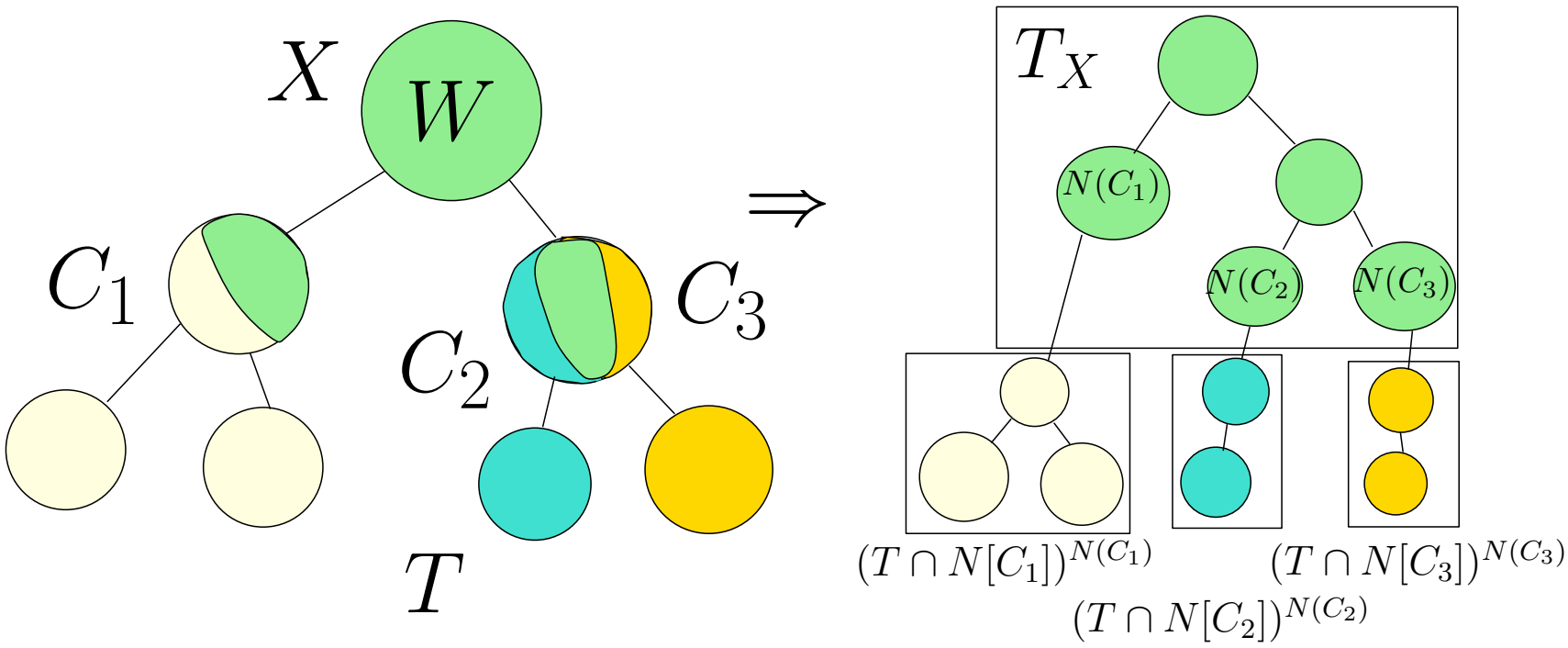

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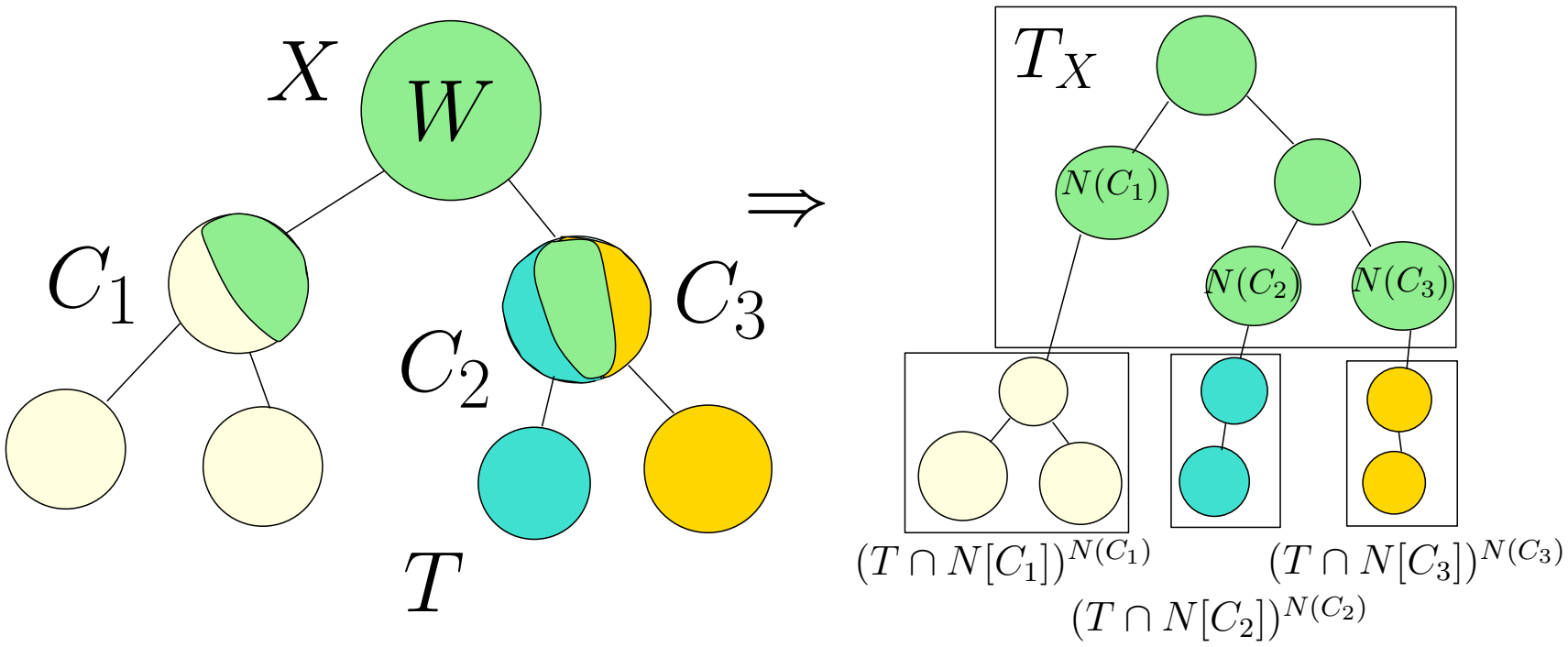

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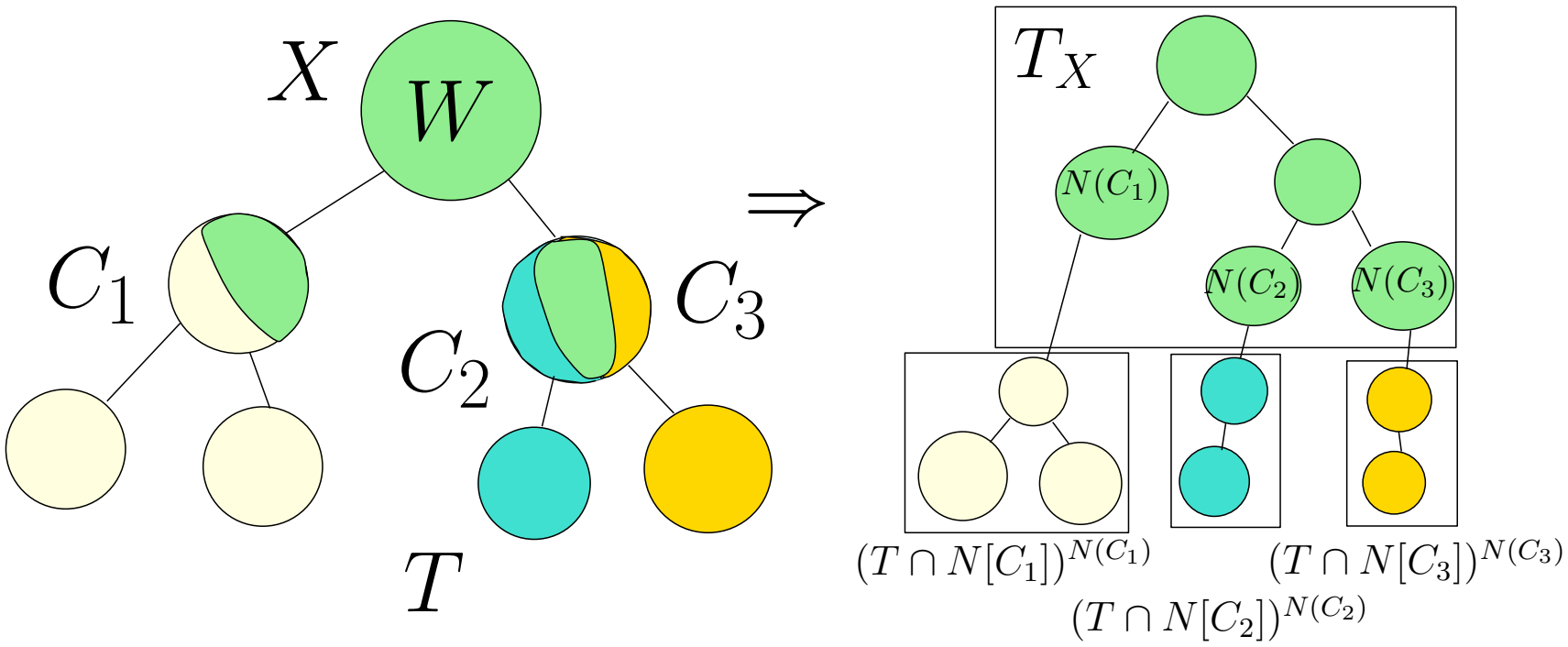

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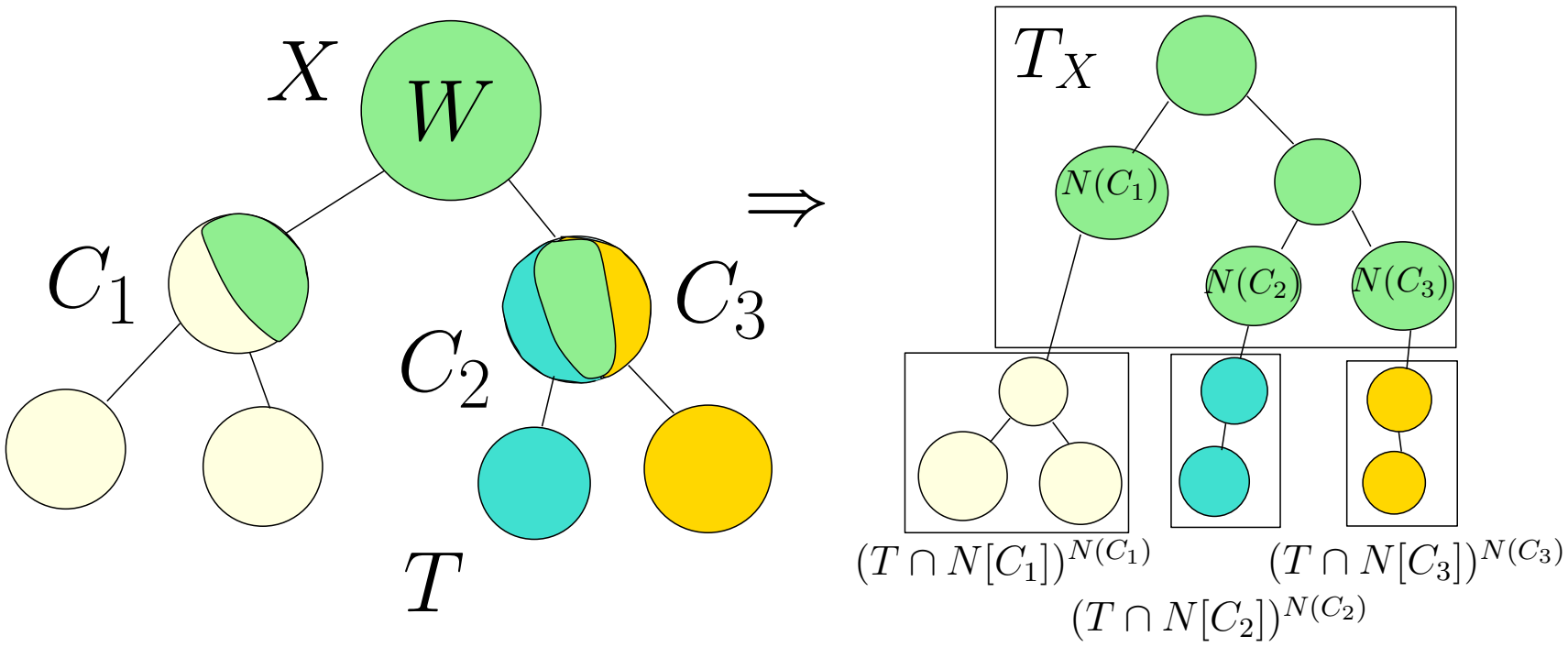

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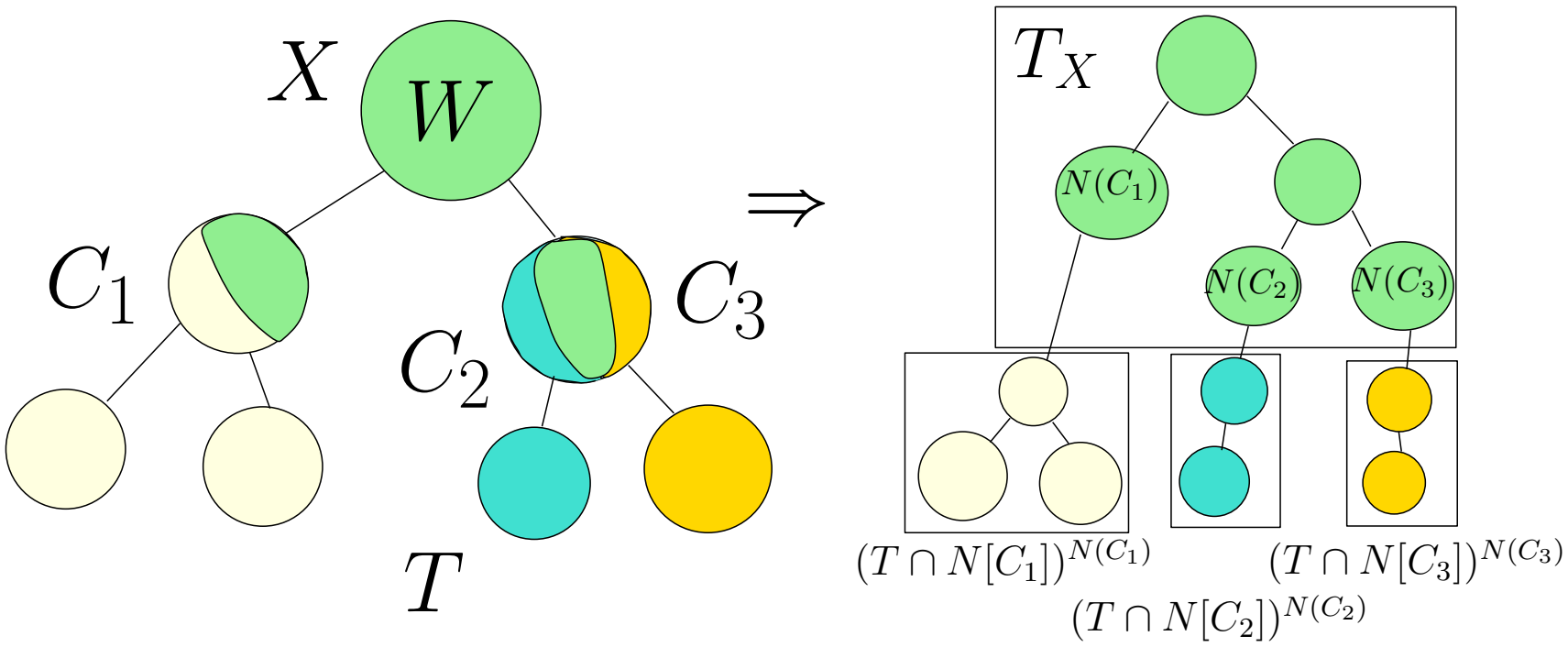

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- (actually needs a bit stronger condition than linkedness for improvement)

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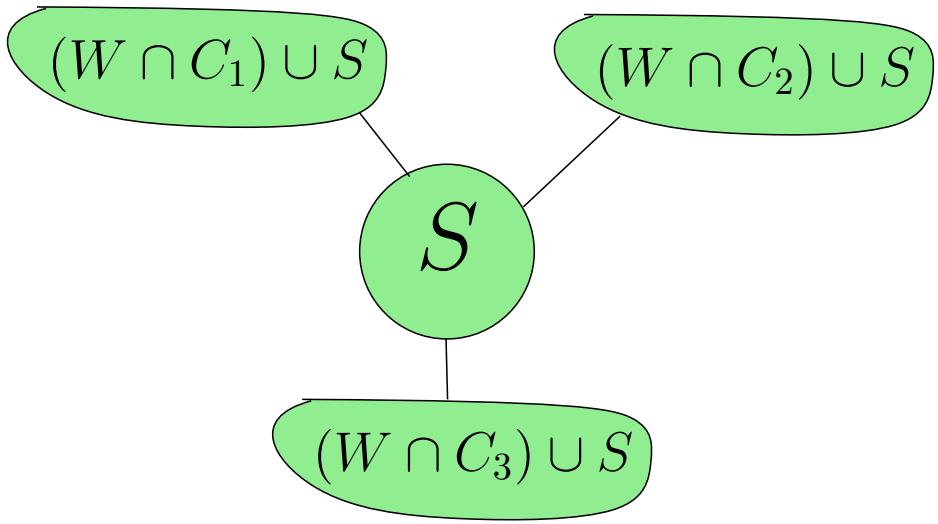
If $|W| \geq 2 \cdot \text{tw}(G) + 3$, then such X and a decomposition of shape $K_{1,3}$ exists



Subset treewidth for 2-approximation

Let W be a largest bag of T SUBSET TREEWIDTH

Want to find:

- a set X with $W \subseteq X \subseteq V(G)$, and
- a tree decomposition of $\texttt{torso}(X)$ of width $\leq |W| - 2$

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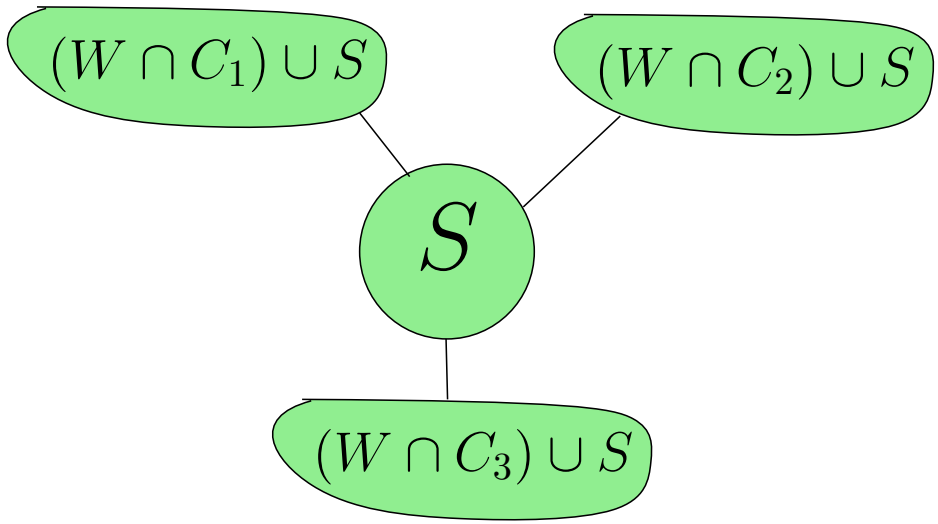

- Where (S, C_1, C_2, C_3) is a balanced 3-way separation of G and $X = S \cup W$

Subset treewidth for exact algorithms

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SUBSET TREEWIDTH

Input: Graph G, integer k, set of vertices $W \subseteq V(G)$ with $|W| = k + 2$

Output: Set $X \subseteq V(G)$ with $W \subseteq X$ and tree decomposition of $\texttt{torso}(X)$ of width $\leq k$ or that the treewidth of G is $> k$

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Theorem

If there is an $f(k) \cdot n^{\mathcal{O}(1)}$ time algorithm for subset treewidth, then there is an $f(k) \cdot n^{\mathcal{O}(1)}$ time algorithm for treewidth with the same function f.

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$2^{\mathcal{O}(k^2)} n^2$ time algorithm for subset treewidth $\rightarrow$ $2^{\mathcal{O}(k^2)} n^4$ time algorithm for treewidth

Subset treewidth for approximation schemes

Subset treewidth for approximation schemes

PARTITIONED SUBSET TREEWIDTH

Input: Graph G, integer k, set of vertices $W \subseteq V(G)$ with $|W| = k+2$ that is partitioned into t cliques $W_1, \ldots, W_t$

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If there is an $f(k,t) \cdot n^{\mathcal{O}(1)}$ time algorithm for partitioned subset treewidth, then there is a $f(\mathcal{O}(k), \mathcal{O}(1/\varepsilon)) \cdot k^{\mathcal{O}(k)} n^{\mathcal{O}(1)}$ time $(1+\varepsilon)$-approximation algorithm for treewidth with the same function f.

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$k^{\mathcal{O}(kt)} n^2$ time algorithm for partitioned subset treewidth $\rightarrow$ $k^{\mathcal{O}(k/\varepsilon)} n^4$ time $(1+\varepsilon)$-approximation algorithm for treewidth

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- Idea: Can afford to increase treewidth by εk
- Any set W can be partitioned into $t = \mathcal{O}(1/\varepsilon)$ sets $W_1, \ldots, W_t$ so that making them into cliques increases treewidth by at most $\varepsilon|W|$

Solving the subset treewidth problem

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Goal: Sketch $k^{\mathcal{O}(kt)}n^{\mathcal{O}(1)}$ time algorithm for partitioned subset treewidth

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(this is also a $k^{\mathcal{O}(k^2)}n^{\mathcal{O}(1)}$ time algorithm for subset treewidth)

Solving subset treewidth

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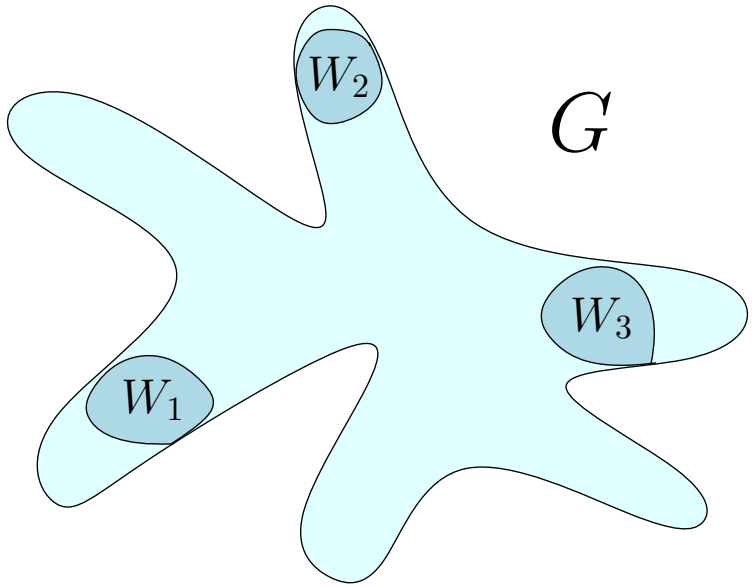

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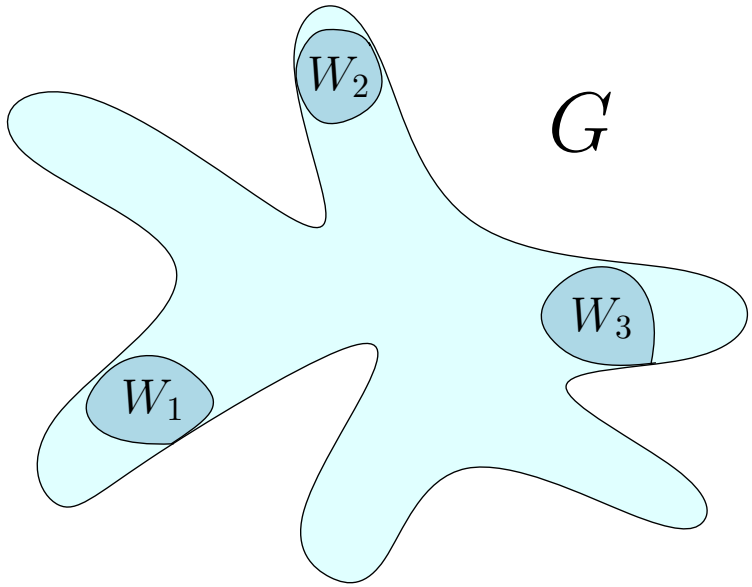

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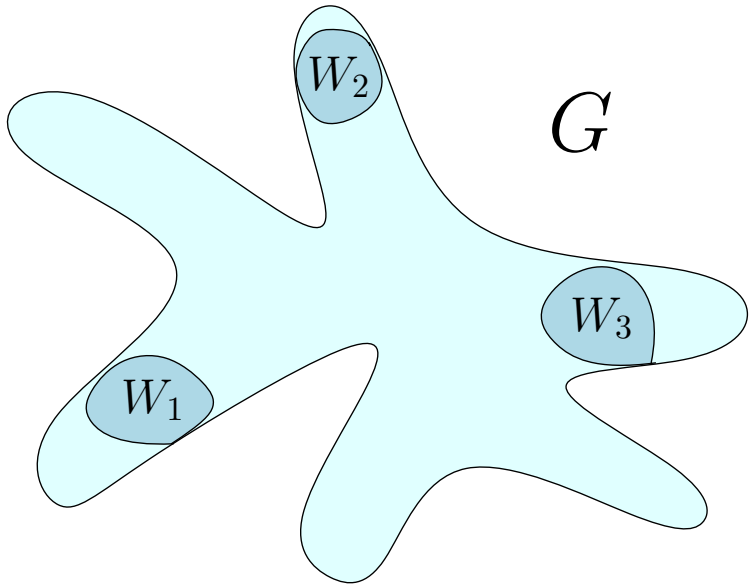

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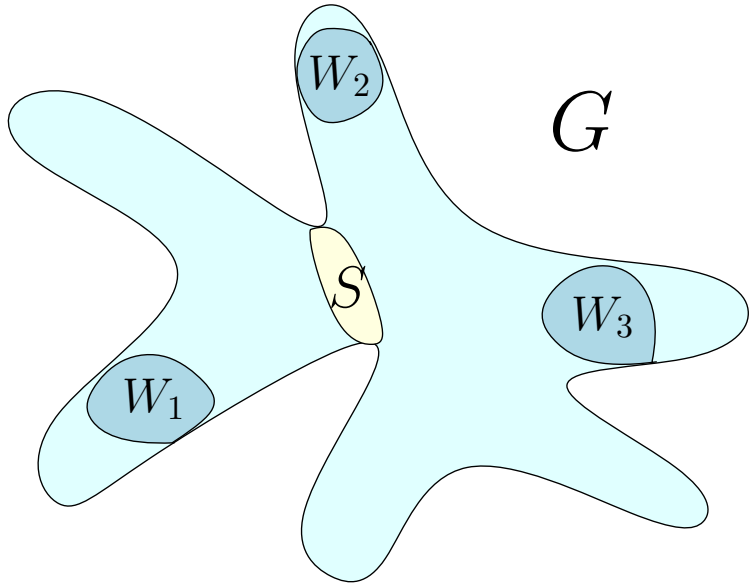
Let S be a non-trivial minimum size (W_i, W_j)-separator



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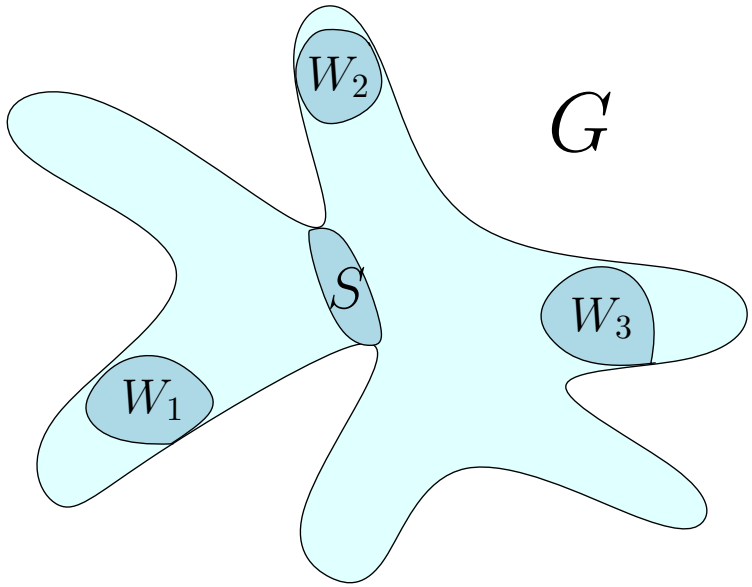
Make S into a terminal clique and solve both sides independently



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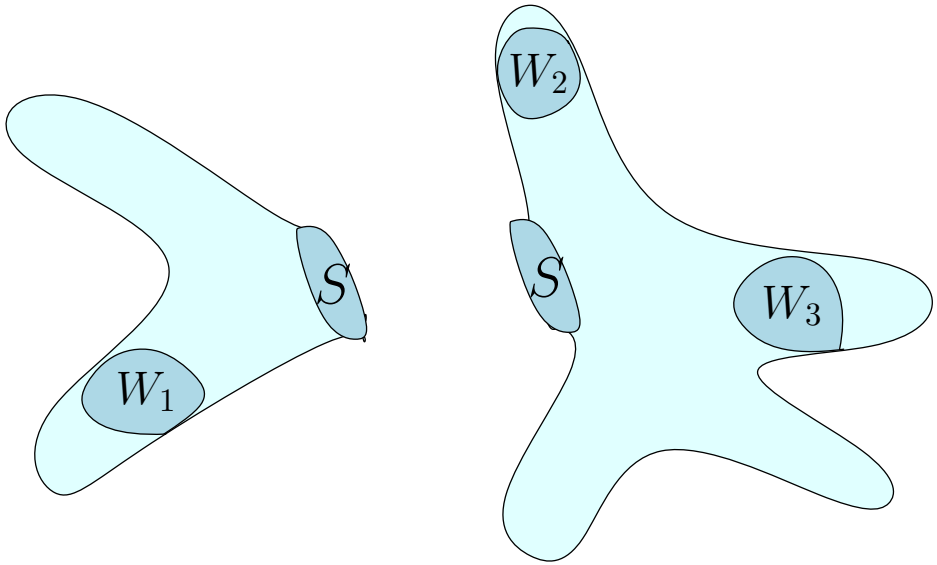
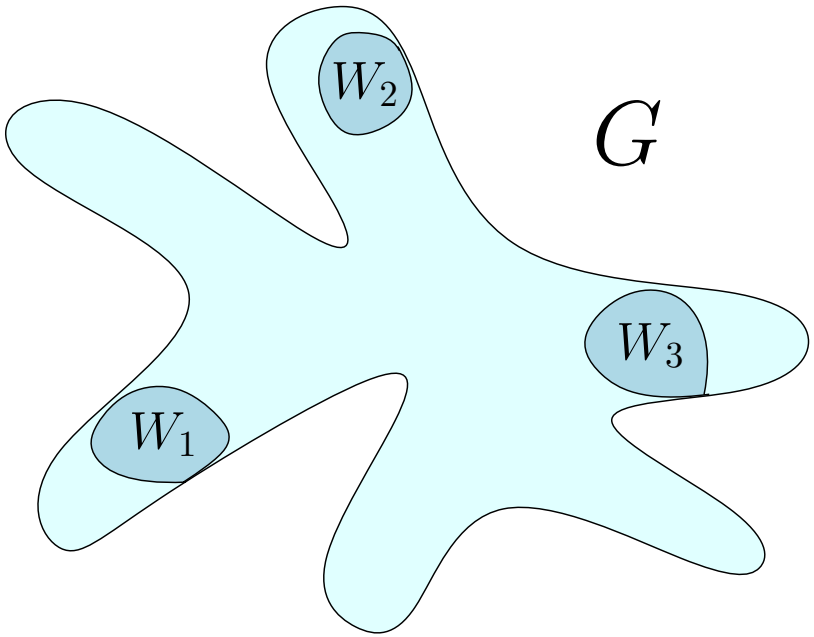


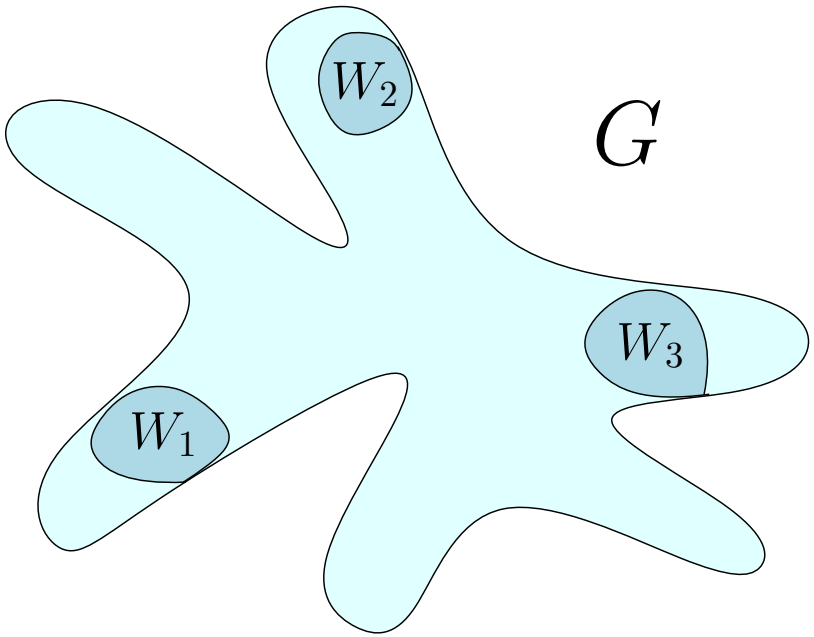
Branching for partitioned subset treewidth

- Now terminal cliques *strongly linked* into each other



Branching for partitioned subset treewidth

- Now terminal cliques *strongly linked* into each other
- Goal: To make progress, increase the size/flow of some terminal clique



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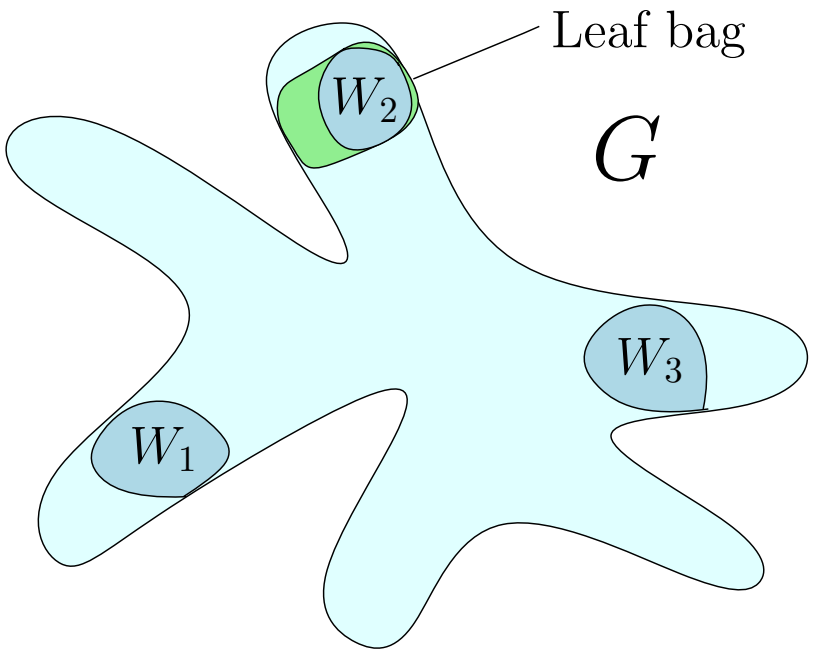

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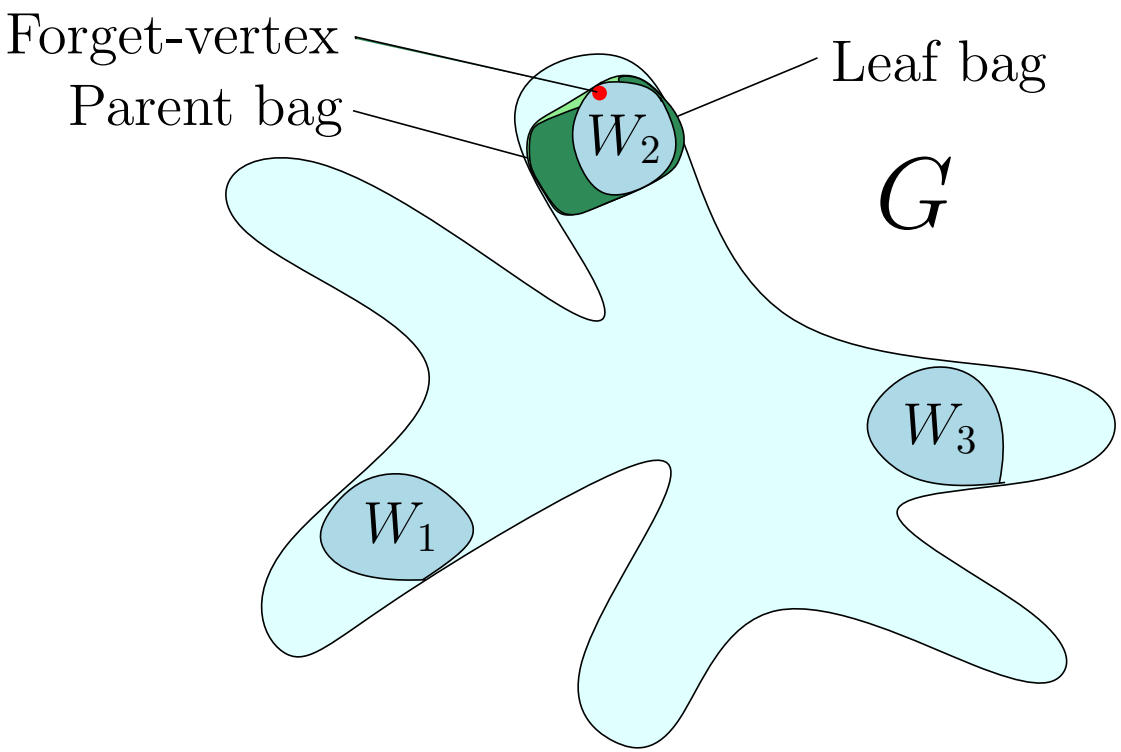

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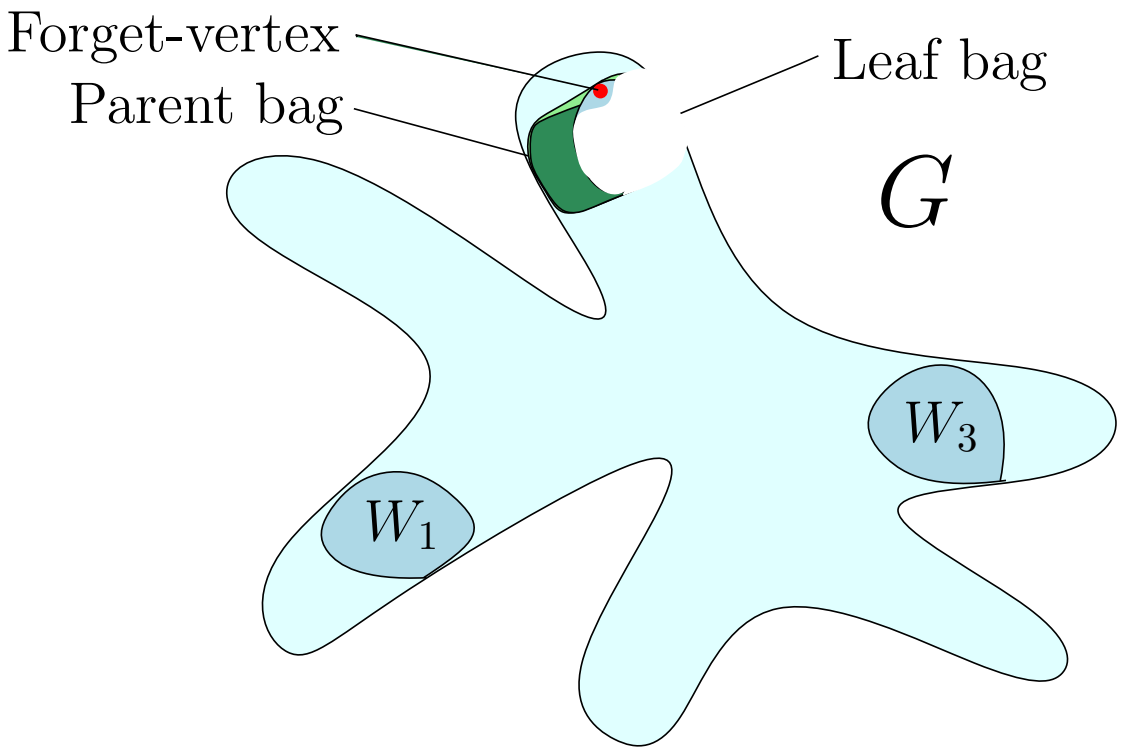

Branching for partitioned subset treewidth

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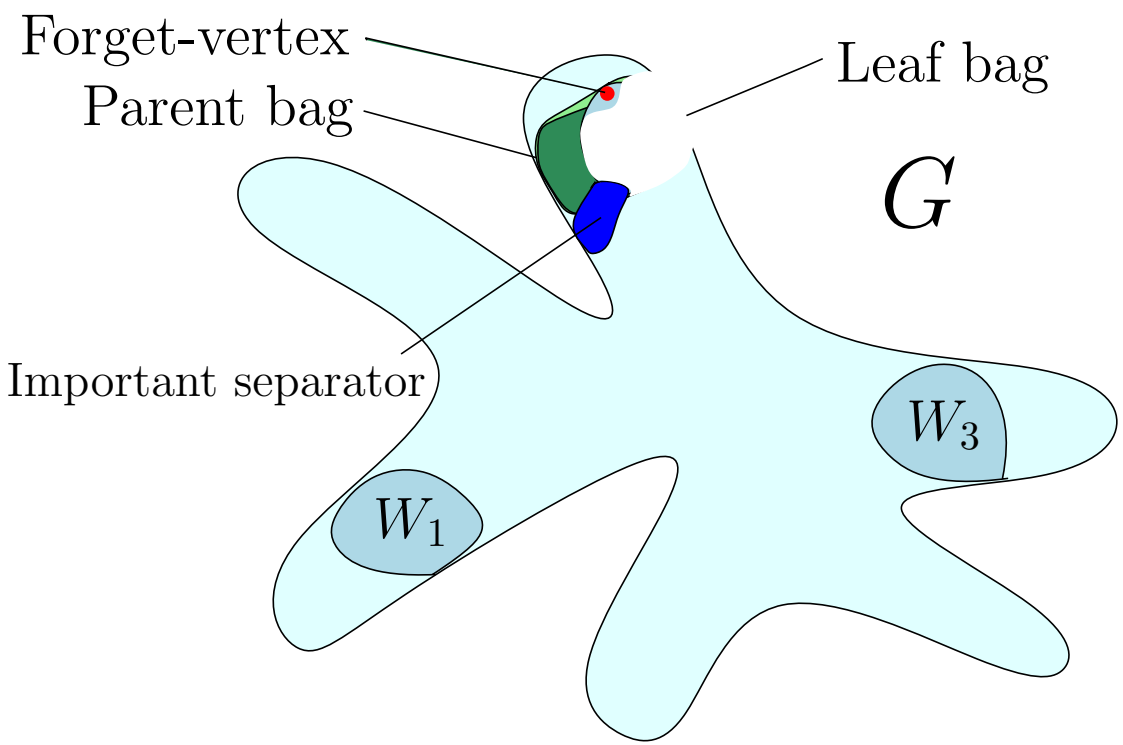
- Goal: To make progress, increase the size/flow of some terminal clique
- Increase W_2 by guessing an important separator



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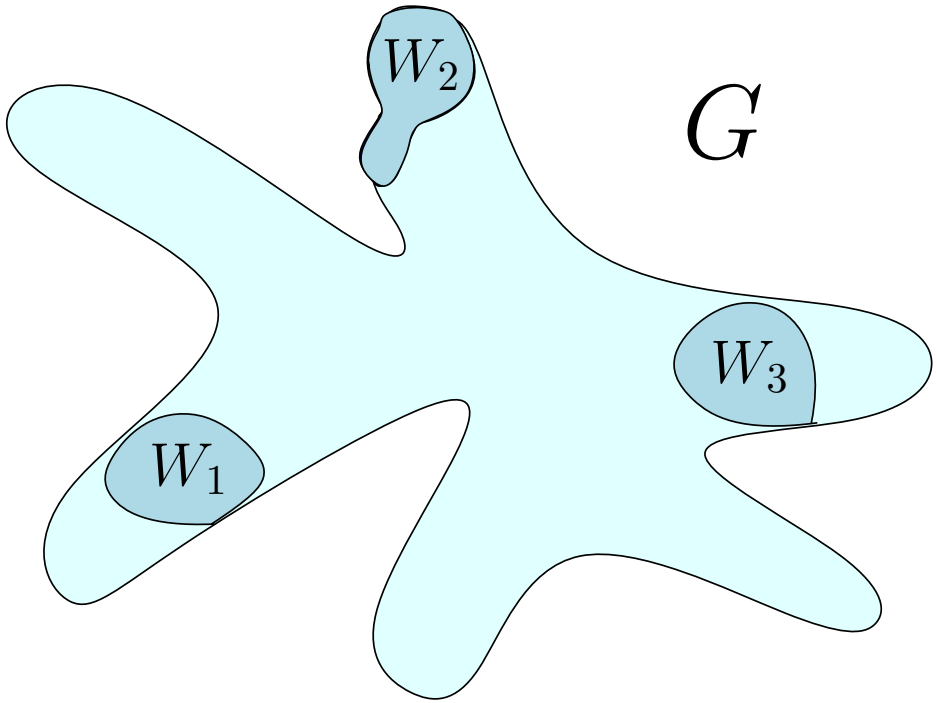
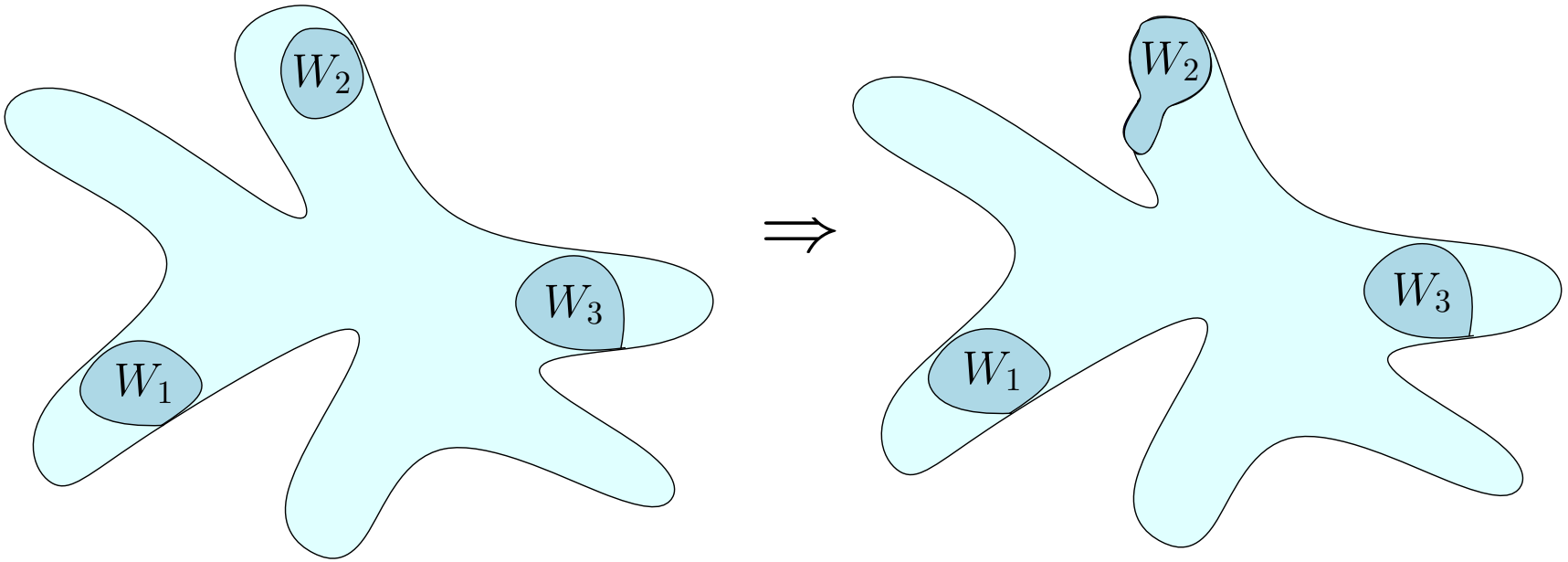


Analysis of branching



- Increased the size/flow of a leaf terminal clique by guessing a forget-vertex and an important separator

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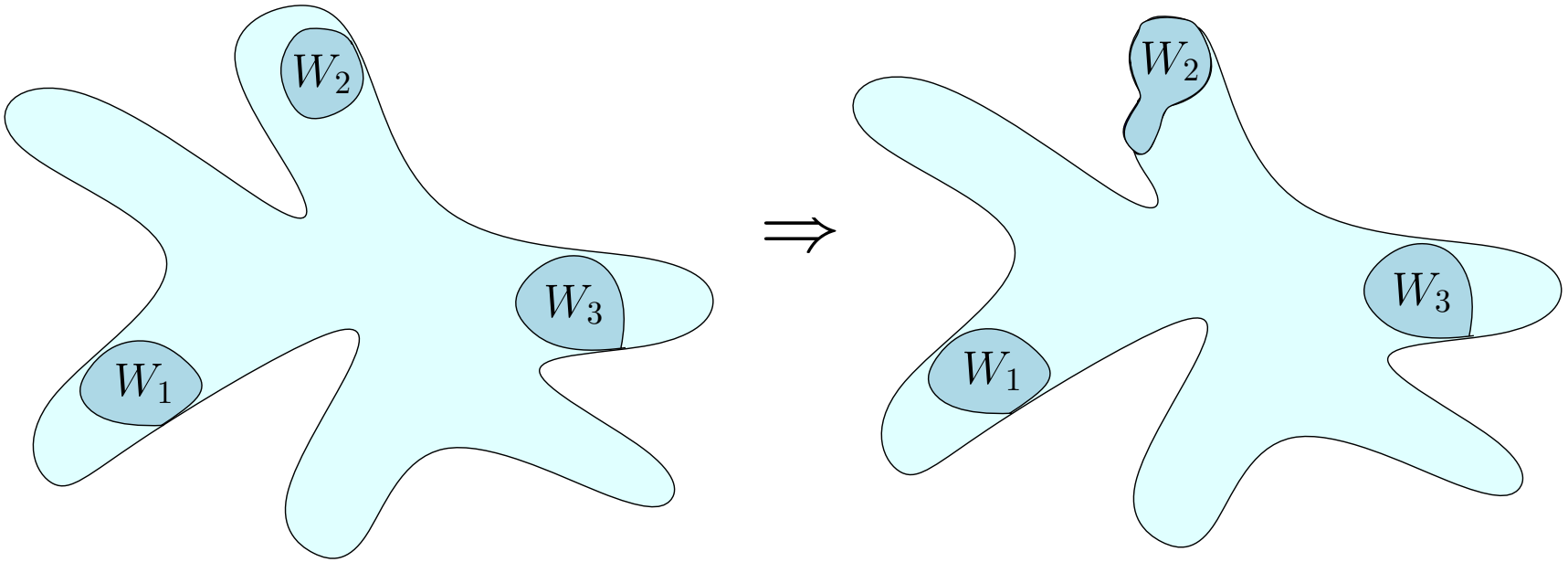
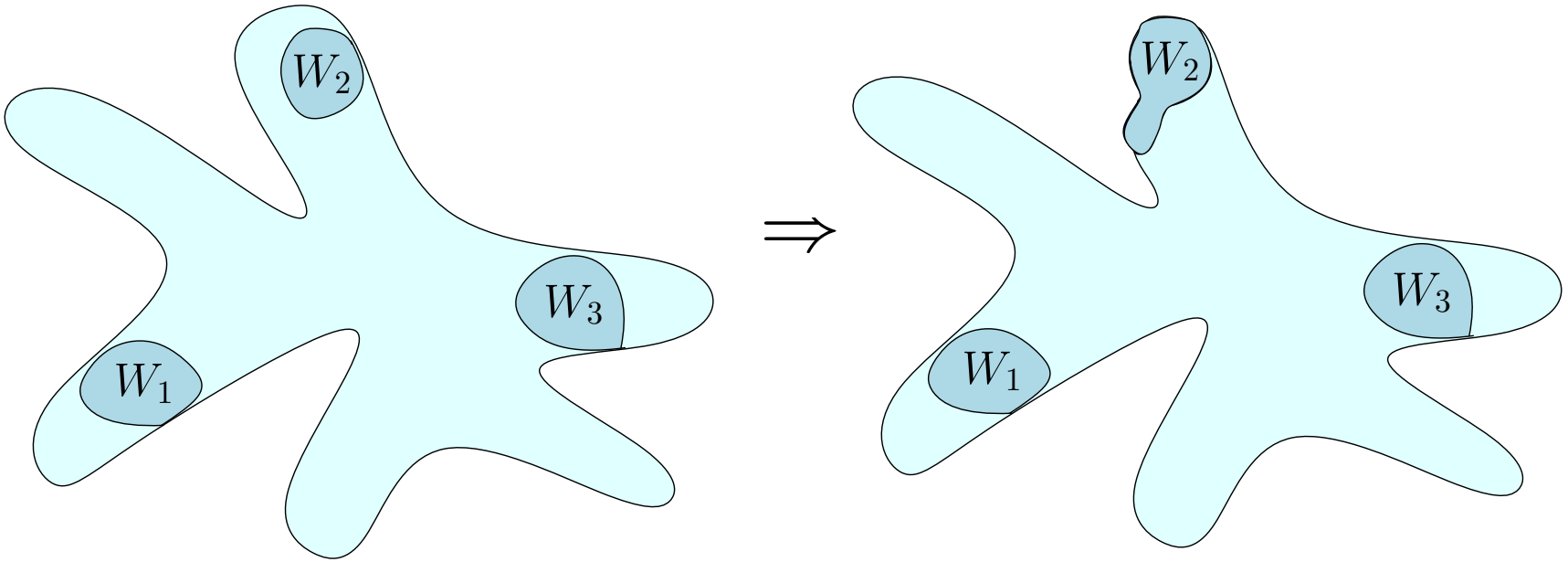


- Increased the size/flow of a leaf terminal clique by guessing a forget-vertex and an important separator
- Sum of sizes/flows of terminal cliques at most $(k+1)t$, so branching depth at most kt

Analysis of branching



- Increased the size/flow of a leaf terminal clique by guessing a forget-vertex and an important separator
- Sum of sizes/flows of terminal cliques at most $(k+1)t$, so branching depth at most kt
- To get $k^{\mathcal{O}(kt)}n^{\mathcal{O}(1)}$ time, need also an important separator hitting set lemma

Conclusion

Open questions:

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- Is there $2^{\mathcal{O}(k^{1.999})}n^{\mathcal{O}(1)}$ time algorithm for subset treewidth?

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Conclusion

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- Is there $2^{\mathcal{O}(k^{1.999})}n^{\mathcal{O}(1)}$ time algorithm for subset treewidth?
- When $t = \mathcal{O}(1)$, is there $2^{\mathcal{O}(k)}n^{\mathcal{O}(1)}$ time algorithm for partitioned subset treewidth?
- How much can the n^4 factor be optimized?

Thank you!

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