Dynamic Treewidth

Tuukka Korhonen[1], Konrad Majewski[2], Wojciech Nadara[2],
Michał Pilipczuk[2], and Marek Sokołowski[2]

[1]University of Bergen, [2]University of Warsaw

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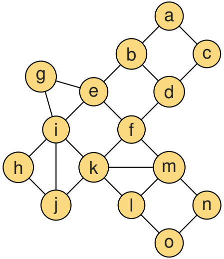
HALG 2024

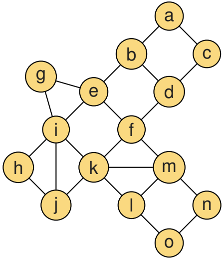
5 June 2024

Treewidth



Graph G

Treewidth



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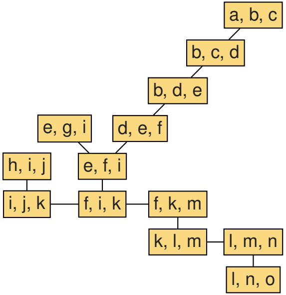

A tree decomposition of G

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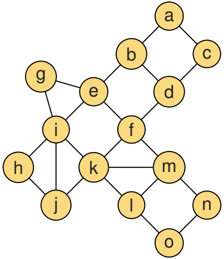

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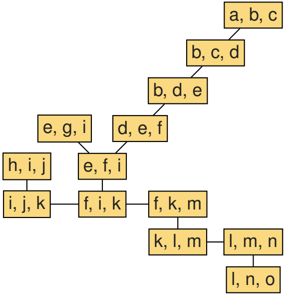

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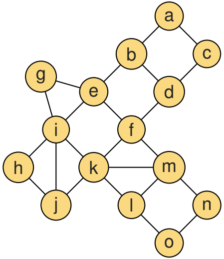

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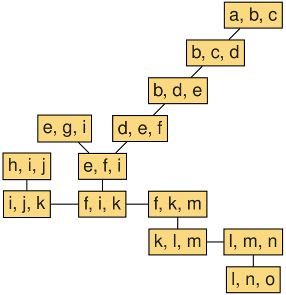

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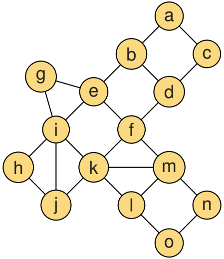

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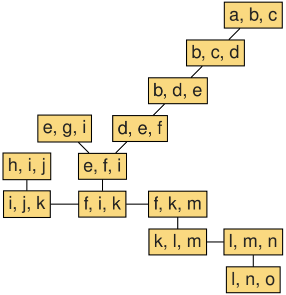

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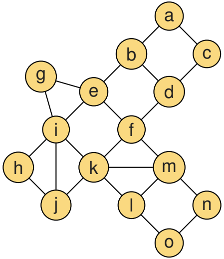

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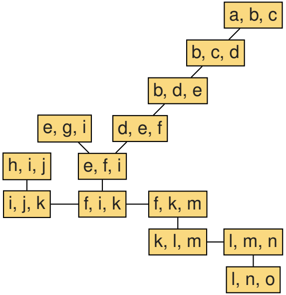

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1. Every vertex should be in a bag
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4. Width = maximum bag size -1

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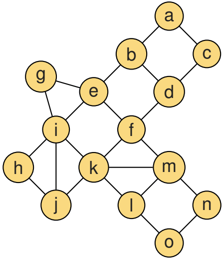

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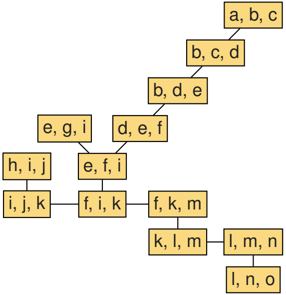

A tree decomposition of G
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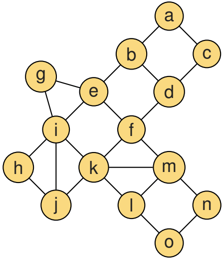

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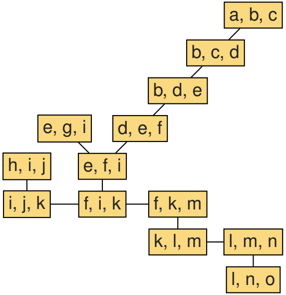

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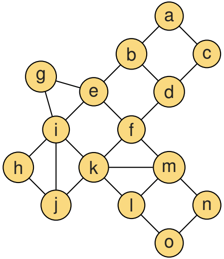

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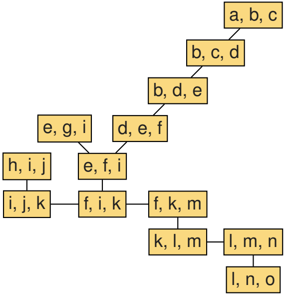

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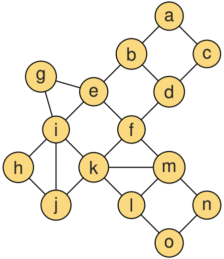

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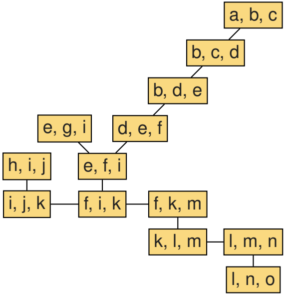

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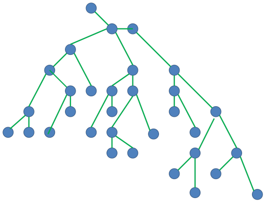
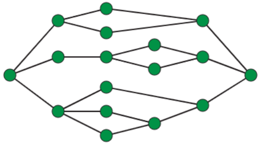
[Robertson & Seymour'84, Arnborg & Proskurowski'89, Bertele & Brioschi'72, Halin'76]

Treewidth of graphs

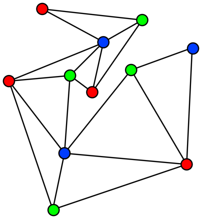
Some graphs of small treewidth:



Trees (tw $\leq$ 1)



Series-parallel (tw $\leq$ 2)



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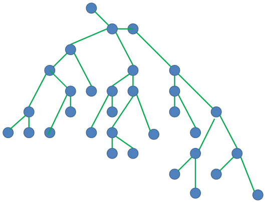

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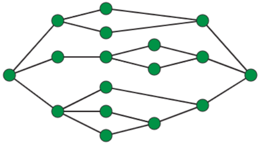

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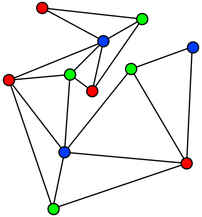

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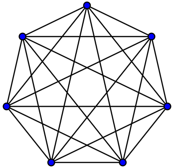
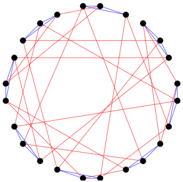
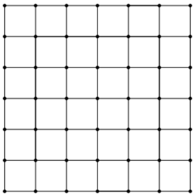


Clique (tw $= n - 1$)



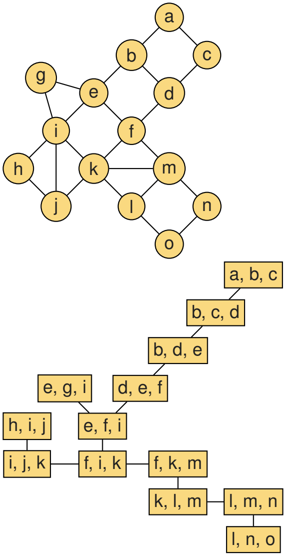
Expanders (tw $= \Theta(n)$)



$n \times m$-grid (tw $= \min(n, m)$)

Why treewidth?

- Algorithms for trees often generalize to algorithms for graphs of small treewidth



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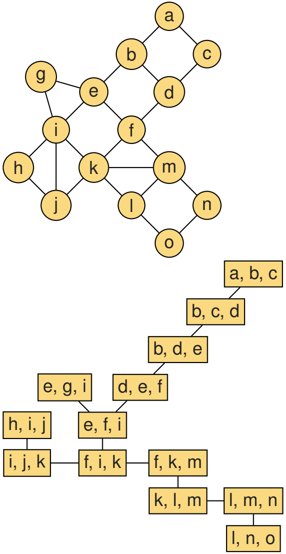

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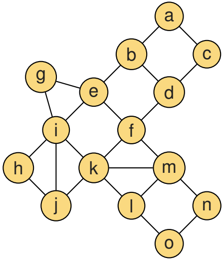

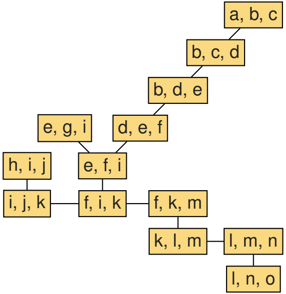

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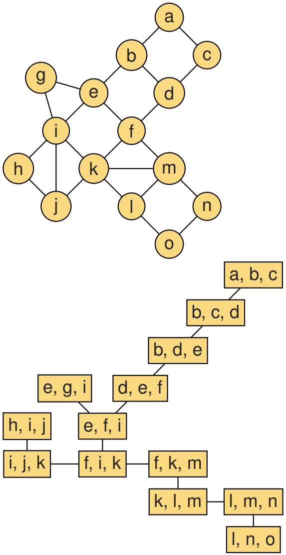

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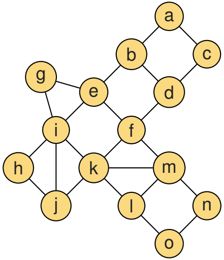
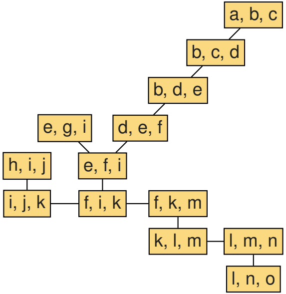
  - $\mathcal{O}_k(n)$ time algorithm to compute an optimum-width tree decomposition [Bodlaender '96]





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- [Goranci, Räcke, Saranurak & Tan '21]: $n^{o(1)}$ amortized time $n^{o(1)}$-approximate tree decomposition on bounded-degree graphs. Not suitable for dynamic programming.

Our result

Summary of previous results

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Theorem (this work):

There is data structure that

- is initialized with integer k and empty n-vertex graph G
- supports edge insertions and deletions in amortized time $\mathcal{O}_k(2^{\sqrt{\log n} \log \log n}) = \mathcal{O}_k(n^{o(1)})$ under the promise that the treewidth of G never exceeds k
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- maintains a tree decomposition of G of width at most $6k + 5$
- can also maintain any dynamic programming scheme on the decomposition within similar running time (formalized by tree-automata)

The algorithm

High-level plan

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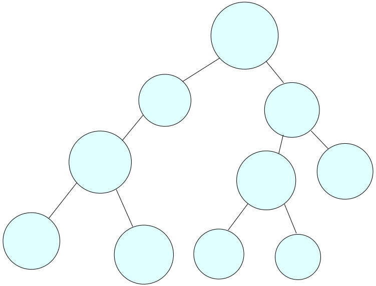
- Goal: Maintain a rooted binary tree decomposition of width $6k + 5$ and depth $d = 2^{\mathcal{O}_k(\sqrt{\log n \log \log n})}$



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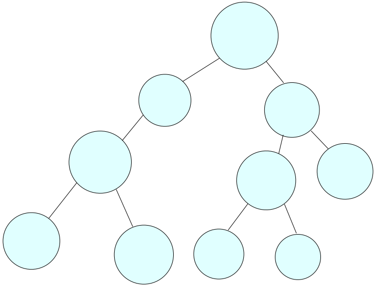

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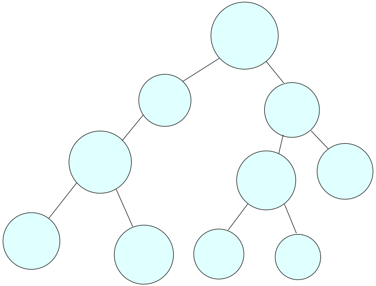

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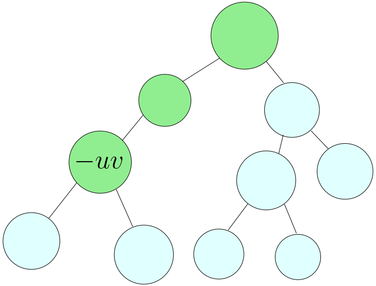
- Edge deletion: Re-compute dynamic programming tables in time $\mathcal{O}_k(d)$



High-level plan

- Goal: Maintain a rooted binary tree decomposition of width $6k + 5$ and depth $d = 2^{\mathcal{O}_k(\sqrt{\log n \log \log n})}$
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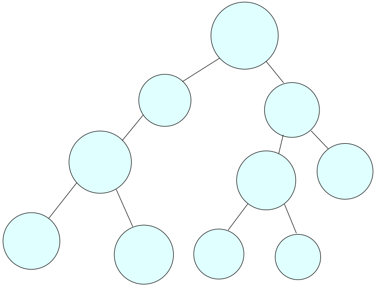

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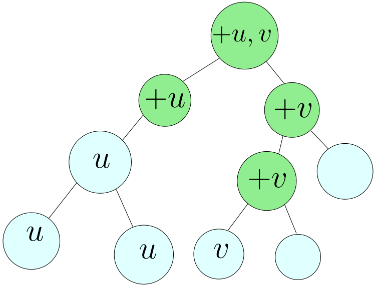

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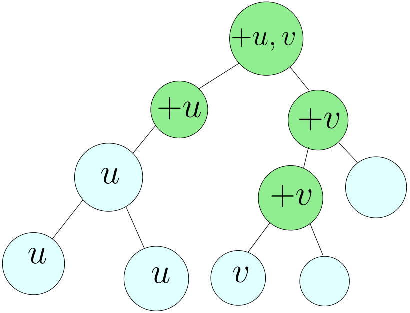

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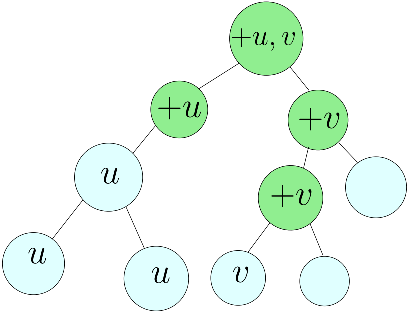
- The width can become more than $6k + 5$ on the green bags!



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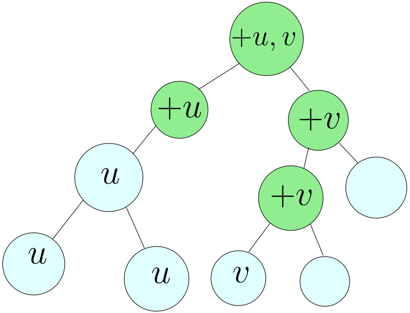
- The width can become more than $6k + 5$ on the green bags!
- Solution: a *Refinement operation* to re-compute the tree decomposition on these bags



Refinement operation

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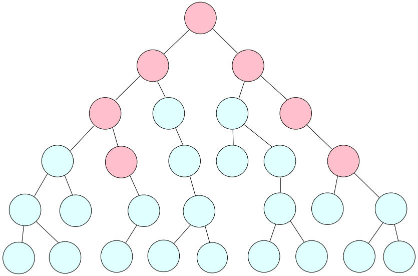

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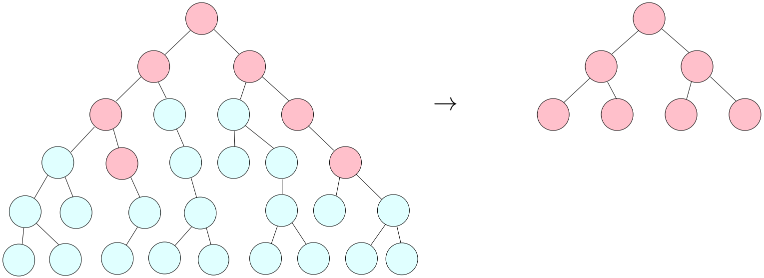

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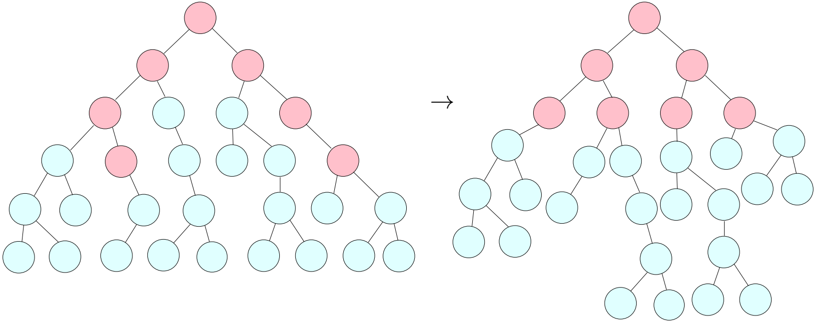

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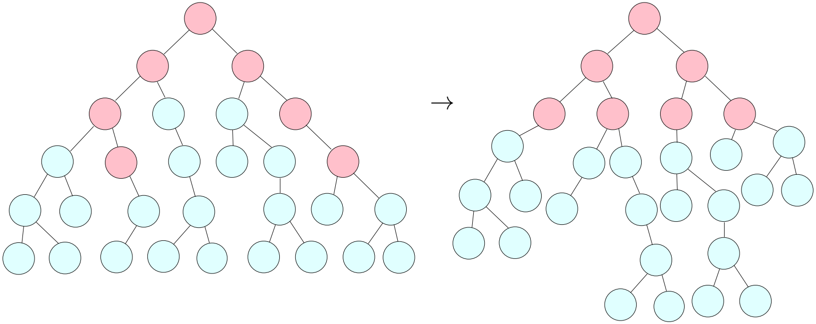

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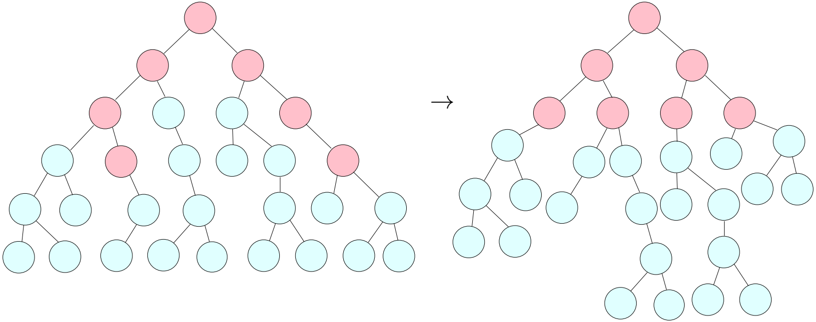
- Builds on the improvement operation of [K. & Lokshtanov'23], also uses the dealternation lemma of [Bojańczyk & Pilipczuk'22] and Bodlaender-Hagerup-lemma



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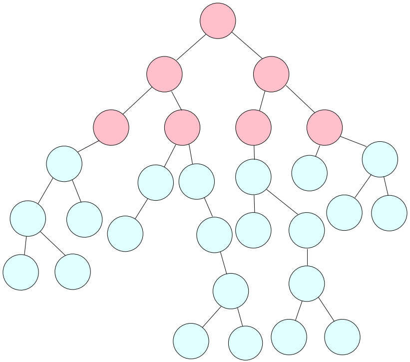

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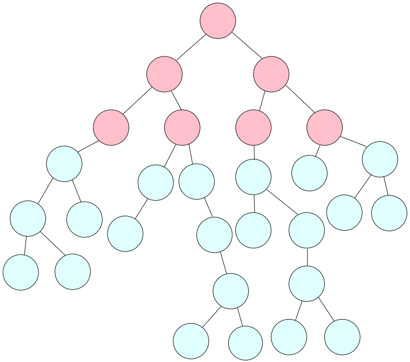

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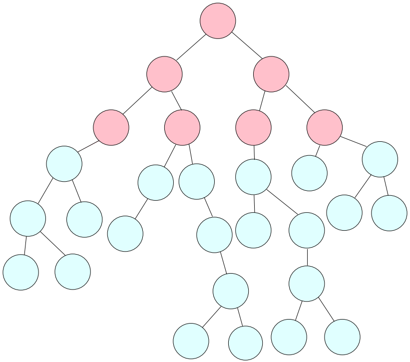

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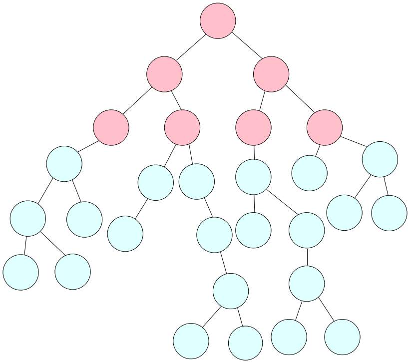
- Solution: A depth-reduction scheme by using the refinement operation and a potential function



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$\Rightarrow$ Can control the height in amortized $2^{\mathcal{O}_k(\sqrt{\log n \log \log n})}$ time

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Thank you!