Dynamic Treewidth

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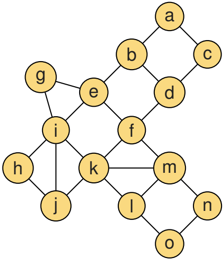
UNIVERSITY OF BERGEN

based on joint work with Konrad Majewski, Wojciech Nadara,
Michał Pilipczuk, and Marek Sokołowski from University of Warsaw

BARC talk

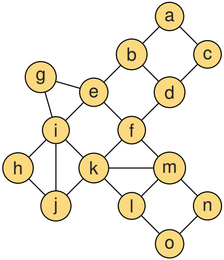
21 November 2023

Treewidth



Graph G

Treewidth



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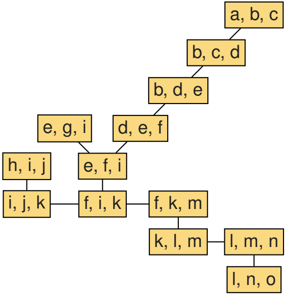
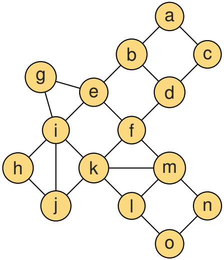


A tree decomposition of G

Treewidth



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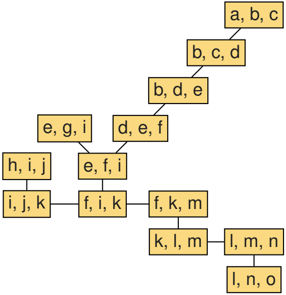

A tree decomposition of G

1. Every vertex should be in a bag

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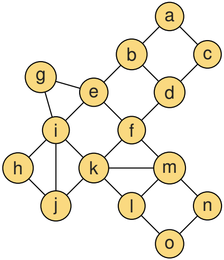

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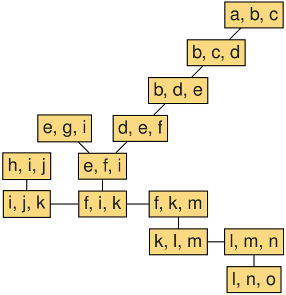

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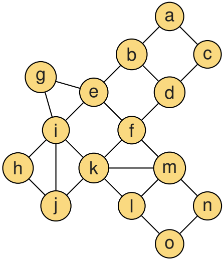

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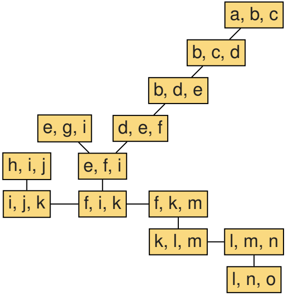

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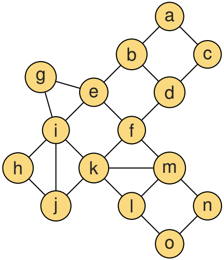

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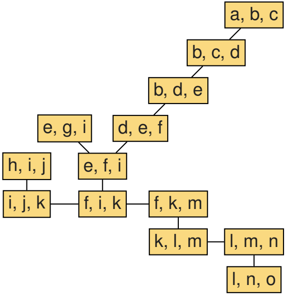

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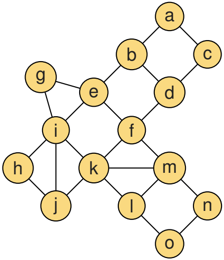

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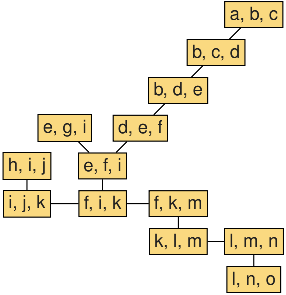

A tree decomposition of G

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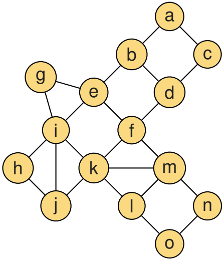

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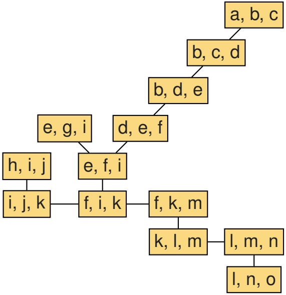

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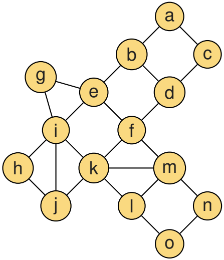

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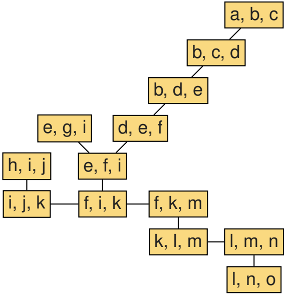

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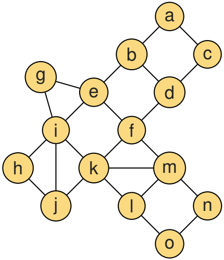

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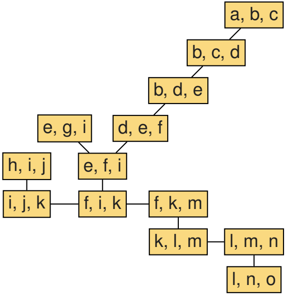

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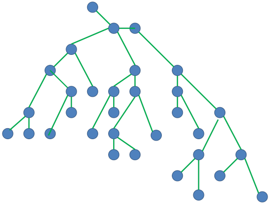
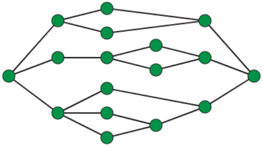
[Robertson & Seymour'84, Arnborg & Proskurowski'89, Bertele & Brioschi'72, Halin'76]

Treewidth of graphs

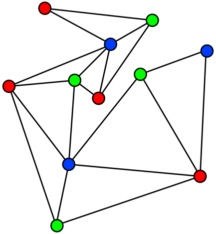
Some graphs of small treewidth:



Trees (tw ≤ 1)



Series-parallel (tw ≤ 2)



k-outerplanar (tw $\leq 3k - 1$)

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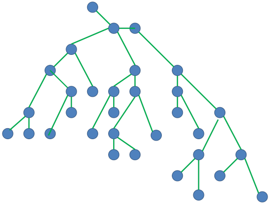

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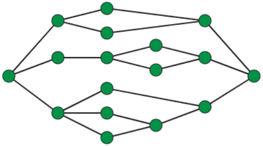

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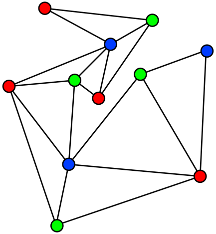

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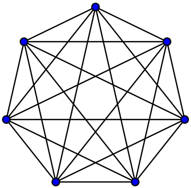
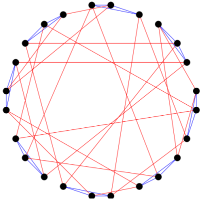
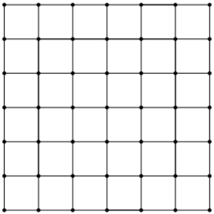


Clique (tw $= n-1$)



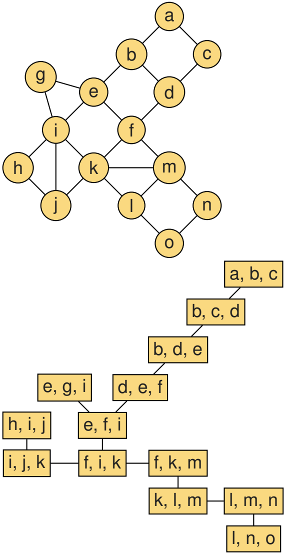
Expanders (tw $= \Theta(n)$)



$n \times m$-grid (tw $= \min(n, m)$)

Why treewidth?

- Algorithms for trees often generalize to algorithms for graphs of small treewidth



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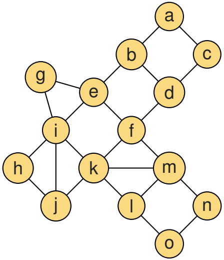

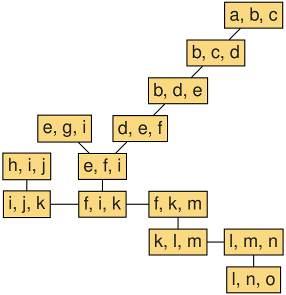

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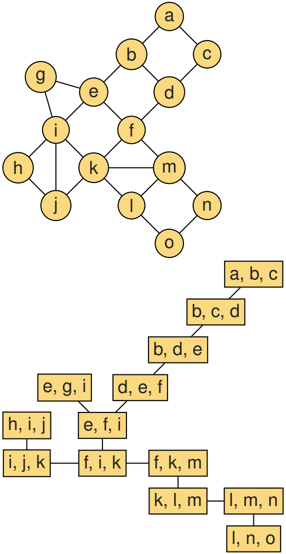
- Courcelle's theorem gives $f(k) \cdot n$ algorithms for all problems definable in **MSO**-logic



Computing treewidth

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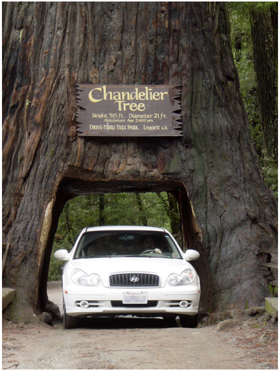

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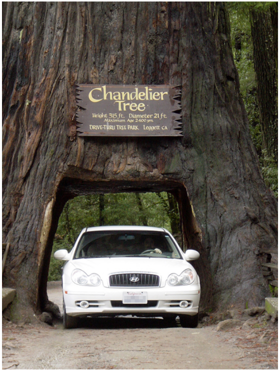

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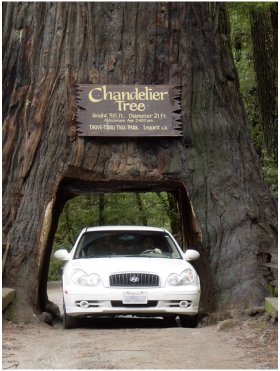

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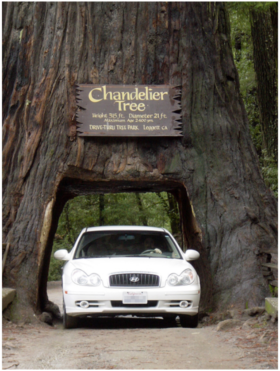
- $\mathcal{O}(\sqrt{\log k})$-approximation in polynomial-time [Feige, Hajiaghayi & Lee '08]



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- [Goranci, Räcke, Saranurak & Tan '21]: $n^{o(1)}$ amortized time $n^{o(1)}$-approximate tree decomposition on bounded-degree graphs. Not suitable for dynamic programming.

Our result

Summary of previous results

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Theorem (this work):

There is data structure that

- is initialized with integer k and empty n-vertex graph G
- supports edge insertions and deletions in amortized time $f(k) \cdot 2^{\sqrt{\log n} \log \log n}$ under the promise that the treewidth of G never exceeds k
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- maintains a tree decomposition of G of width at most $6k + 5$
- can also maintain any dynamic programming scheme on the decomposition within similar running time (formalized by tree-automata)

The algorithm

High-level plan

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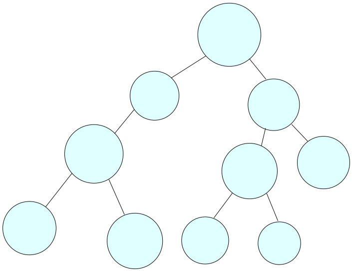
- Goal: Maintain a rooted binary tree decomposition of width $6k+5$ and depth $d = 2^{\mathcal{O}_k(\sqrt{\log n \log \log n})}$



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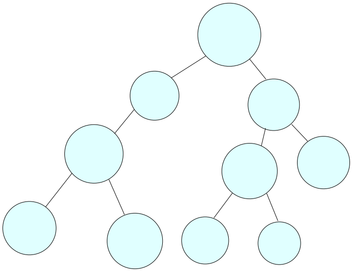

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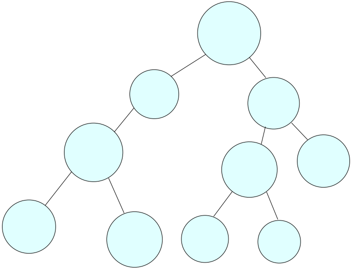

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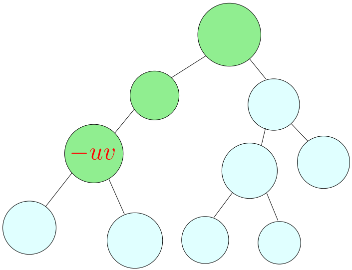

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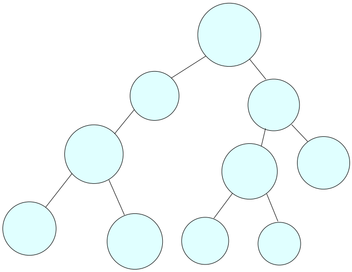

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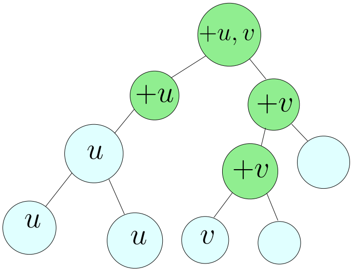
- Maintain also dynamic programming tables directed towards the root
- Edge deletion: Re-compute dynamic programming tables in time $\mathcal{O}_k(d)$
- Edge insertion: Add u and v to all bags on the path from their subtrees to the root, and re-compute dynamic programming tables in time $\mathcal{O}_k(d)$



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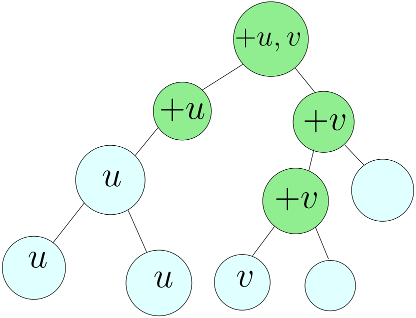

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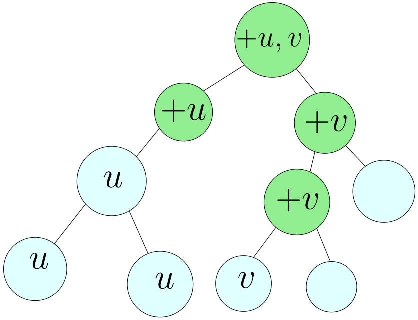

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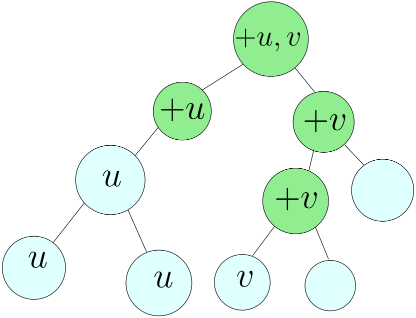
- The width can become more than $6k + 5$ on the green bags!
- Solution: a *Refinement operation* to re-compute the tree decomposition on these bags



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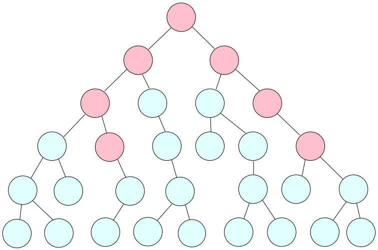

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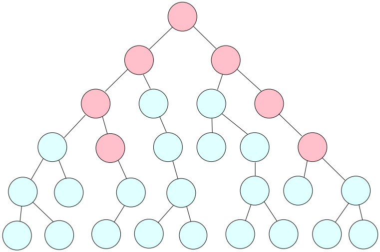

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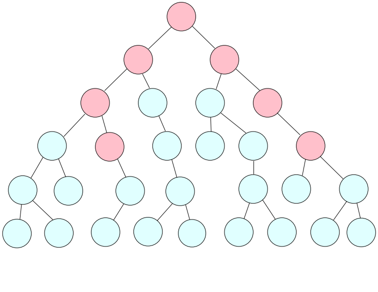

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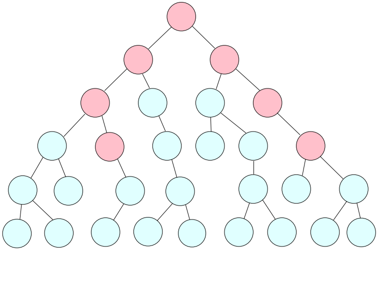

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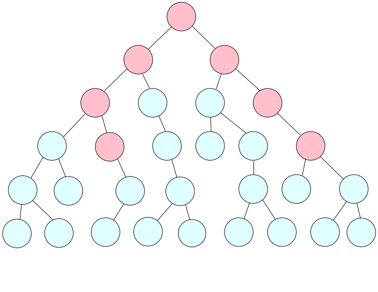
- Builds on the improvement operation of [**K.**&Lokshtanov'23], also uses the dealternation lemma of [Bojańczyk&Pilipczuk'22] and Bodlaender-Hagerup-lemma



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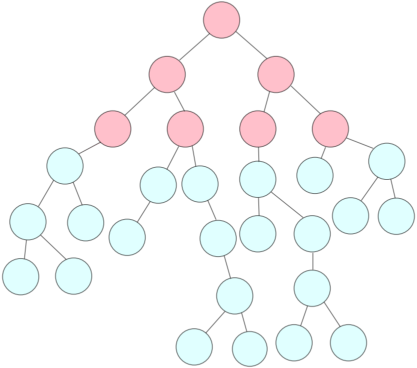

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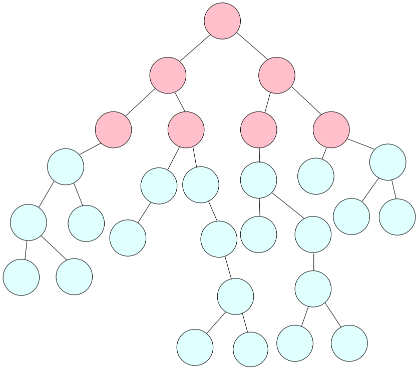

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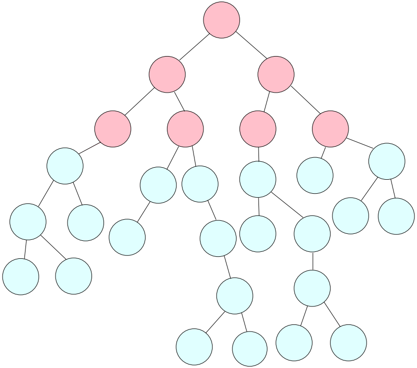

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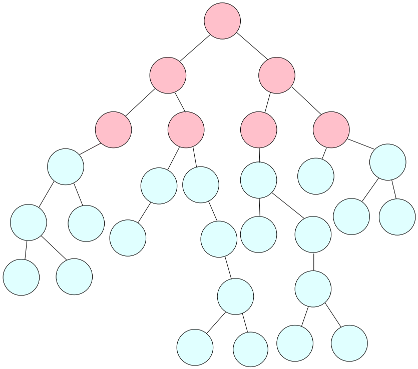
- Solution: A depth-reduction scheme by using the refinement operation and a potential function



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$\Rightarrow$ Can control the height in amortized $2^{\mathcal{O}_k(\sqrt{\log n \log \log n})}$ time

Lemma on trees

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Let $c \geq 2$ and T be a binary tree width n nodes. If the depth of T is at least $2^{\Omega(\sqrt{\log n \log c})}$ then there exists a prefix P of T so that

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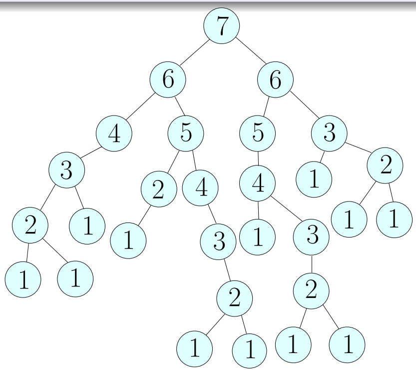

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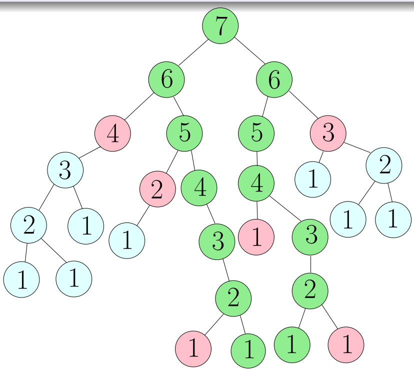

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Thank you!